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CIRCUIT SIMULATION IN MATLAB

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In this paper a circuit simulation of linear electric systems containing independent and dependent sources of DC voltage and/or current is proposed. For circuit simulation is used nodal analysis method and the program based on this method was created in MATLAB environment. The symbolic representation of constraint equations for node voltages and nodal equations for non reference nodes and supernodes is drawn up after setting of prescribed input data and the reference node. Symbolic representation of nodal voltages, branch voltages and branch currents and calculation their numeric values for given circuit element parameters can be also generated.

Key words: circuit simulation, linear electric circuit, nodal analysis, incidence matrix, MATLAB.

МОДЕЛЮВАННЯ ЕЛЕКТРИЧНИХ КІЛ У СЕРЕДОВИЩІ MATLAB

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Розглядається моделювання лінійних електричних систем, які містять незалежні та залежні джерела постійної напруги та/або струму. Для моделювання електричних кіл використовується метод вузлових потенціалів. Програму симуляції, яка базується на використанні даного методу, розроблено в середовищі MATLAB. Символьне надання рівнянь зв'язку для вузлів напруги та вузлових рівнянь для незв'язаних вузлів та «супервузлів» виконується після встановлення прийнятих вхідних даних та вузлів відношень. Також можуть бути розраховані символьне представлення вузлових напруг, напруг гілок та струмів гілок, а також обчислення їх чисельних значень для заданих параметрів елементу електричного кола.

Ключові слова: моделювання електричних кіл, лінійні електричні кола, вузловий аналіз, матриця подій, MATLAB.

PROBLEM STATEMENT. Circuit simulation is a technique for checking and verifying the design of electrical and electronic circuits and systems before to manufacturing and deployment [1]. It combines three important areas of research: mathematical modeling of the circuit elements, formulation of the circuit equations and techniques for solution of these equations.

This paper deals with formulation of the circuit equations. For circuit simulation on a computer it is necessary to use general-purpose methods which work for all circuits and formulation of the circuit equations must be done in systematic and automatic way for every circuit (large or small).

As the general method for symbolic analysis was used nodal analysis and as the programming environment was selected MATLAB environment.

Nodal analysis is method for solving linear electric circuits. Unknown quantities are nodal voltages. A nodal voltage is a voltage between a node pair, which is formed by a nonference node and a reference node. A node is a point of connection of two or more circuit elements. Because MATLAB (MATrix LABoratory) is a tool for matrix solving of problems, it is necessary to create a matrix formulation of nodal analysis.

We assume a proper linear electric circuit containing N_{u} nodes and N_{v} branches. Let us consider a resistor with resistance $R_k \neq 0 \ \wedge \ R_k \neq \infty$, an ideal current source (independent or dependent) with a current $i_k^{(\mathrm{S})}$ and an ideal voltage source with a voltage $u_k^{(\mathrm{S})}$ are connected in a branch $\mathrm{v}_k \ (k = 1, 2, ..., n_{\mathrm{v}})$ having the node p and node q at each end (Fig. 1).

KCL written in the matrix form is [2]:

$$\mathbf{A}\,\mathbf{I}^{(\mathrm{v})} = \mathbf{0}, \qquad (1)$$

where $\mathbf{I}^{(\mathrm{v})}$ is a matrix of branch currents $i_k^{(\mathrm{v})}$ of order $(N_{\mathrm{v}}, 1)$, $\mathbf{A}$ being an incidence matrix of order $(N_{\mathrm{u}} - 1, N_{\mathrm{v}})$ with elements a_{ij} which express incidence between nodes and branches of circuit and their value is $\{+1, -1, 0\}$.

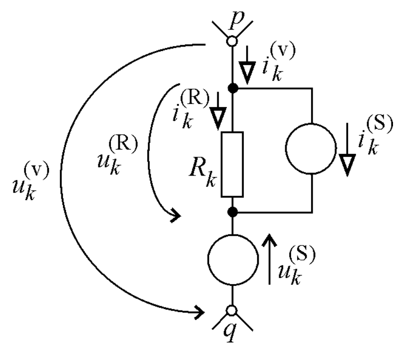

Figure 1 – The *k*th circuit branch connected the node *p* and the node *q*

The branch current $\mathbf{I}^{(\mathrm{v})}$ can be expressed:

$$\mathbf{I}^{(\mathrm{v})} = \mathbf{I}^{(\mathrm{R})} + \mathbf{I}^{(\mathrm{S})}, \qquad (2)$$

where $\mathbf{I}^{(\mathrm{R})}$ is a matrix of resistor currents $i_k^{(\mathrm{R})}$ of order $(N_{\mathrm{v}}, 1)$, $\mathbf{I}^{(\mathrm{S})}$ being a matrix of current source currents $i_k^{(\mathrm{S})}$ of order $(N_{\mathrm{v}}, 1)$.

The resistor current $\mathbf{I}^{(\mathrm{R})}$ will be:

$$\mathbf{I}^{(\mathrm{R})} = \mathbf{G}\left(\mathbf{U}^{(\mathrm{v})} + \mathbf{U}^{(\mathrm{S})}\right), \qquad (3)$$

where $\mathbf{G}$ is a diagonal matrix of resistor conductance of order $(N_\mathrm{v}, N_\mathrm{v})$, $\mathbf{U}^{(\mathrm{v})}$ being a matrix of branch voltages of order $(N_\mathrm{v}, 1)$, $\mathbf{U}^{(\mathrm{S})}$ being a matrix of voltage source voltages of order $(N_\mathrm{v}, 1)$.

Each branch voltage can be expressed in terms of a nodal voltage [1]:

$$\mathbf{U}^{(\mathrm{v})} = \mathbf{A}^\mathrm{T}\mathbf{U}^{(\mathrm{n})}, \qquad (4)$$

where $\mathbf{A}^\mathrm{T}$ is a transpose matrix to a matrix $\mathbf{A}$, $\mathbf{U}^{(\mathrm{n})}$ being a matrix of nodal voltages of order $(N_\mathrm{u} - 1, 1)$.

Substituting equations (2), (3) and (4) into equation (1), we obtain

$$\mathbf{A}\,\mathbf{G}\mathbf{A}^\mathrm{T}\mathbf{U}^{(\mathrm{n})} + \mathbf{A}\mathbf{G}\mathbf{U}^{(\mathrm{S})} + \mathbf{A}\mathbf{I}^{(\mathrm{S})} = \mathbf{0}. \qquad (5)$$

The equation (5) represents a matrix formulation of nodal equations in terms of incidence matrix $\mathbf{A}$ for electric circuits containing current and/or voltage sources (independent and dependent).

Matrix $\mathbf{U}^{(\mathrm{S})}$ in the equation (5) is following:

$$\mathbf{U}^{(\mathrm{S})} = \mathbf{U}^{(\mathrm{IVS})} + \mathbf{U}^{(\mathrm{CCVS})} + \mathbf{U}^{(\mathrm{VCVS})}, \qquad (6)$$

where $\mathbf{U}^{(\mathrm{IVS})}$ is a matrix of voltages independent voltage sources with a positive sign if its polarity is not oriented corresponding to a branch orientation, $\mathbf{U}^{(\mathrm{CCVS})}$ being a matrix of voltages of current-controlled voltage sources (CCVS), $\mathbf{U}^{(\mathrm{VCVS})}$ being a matrix of voltages of voltage-controlled voltage sources (VCVS).

Matrix $\mathbf{U}^{(\mathrm{CCVS})}$ in the equation (6) is following:

$$\mathbf{U}^{(\mathrm{CCVS})} = \mathbf{V}^{(\mathrm{CCVS})}\mathbf{I}^{(\mathrm{v})}, \qquad (7)$$

where $\mathbf{V}^{(\mathrm{CCVS})}$ is a matrix of coupling coefficients between voltages CCVS and controlling currents with a positive sign if its polarity is not oriented corresponding to a orientation of branch in which is the source, and simultaneous a polarity of controlling current is corresponding to a orientation of branch in which it is placed.

Matrix $\mathbf{U}^{(\mathrm{VCVS})}$ in the equation (6) is following:

$$\mathbf{U}^{(\mathrm{VCVS})} = \mathbf{V}^{(\mathrm{VCVS})}\mathbf{U}^{(\mathrm{v})}, \qquad (8)$$

where $\mathbf{V}^{(\mathrm{VCVS})}$ is a matrix of coupling coefficients between voltages CCVS and controlling voltages with a positive sign if its polarity is not oriented corresponding to an orientation of branch in which is the source, and simultaneous a polarity of controlling voltage is oriented corresponding to a orientation of branch in which it is placed.

Matrix $\mathbf{I}^{(\mathrm{S})}$ in the equation (5) is following:

$$\mathbf{I}^{(\mathrm{S})} = \mathbf{I}^{(\mathrm{ICS})} + \mathbf{I}^{(\mathrm{CCCS})} + \mathbf{I}^{(\mathrm{VCCS})}. \qquad (9)$$

where $\mathbf{I}^{(\mathrm{ICS})}$ is a matrix of currents independent current sources with a positive sign if its polarity is oriented corresponding to a branch orientation, $\mathbf{I}^{(\mathrm{CCCS})}$ being a matrix of currents of current-controlled current sources (CCCS), $\mathbf{I}^{(\mathrm{VCCS})}$ being a matrix of currents o voltage-controlled current sources (VCCS).

Matrix $\mathbf{I}^{(\mathrm{CCCS})}$ in the equation (9) is following:

$$\mathbf{I}^{(\mathrm{CCCS})} = \mathbf{V}^{(\mathrm{CCCS})}\mathbf{I}^{(\mathrm{v})}, \qquad (10)$$

where $\mathbf{V}^{(\mathrm{CCCS})}$ is a matrix of coupling coefficients between currents CCCS and controlling currents with a positive sign if its polarity is oriented corresponding to an orientation of branch in which is the source, and simultaneous a polarity of controlling current is oriented corresponding to a orientation of branch in which it is placed.

Matrix $\mathbf{I}^{(\mathrm{VCCS})}$ in the equation (9) is following:

$$\mathbf{I}^{(\mathrm{VCCS})} = \mathbf{V}^{(\mathrm{VCCS})}\mathbf{U}^{(\mathrm{v})}, \qquad (11)$$

where $\mathbf{V}^{(\mathrm{VCCS})}$ is a matrix of coupling coefficients between currents VCCS and controlling currents with a positive sign if its polarity is oriented corresponding to an orientation of branch in which is the source, and simultaneous a polarity of controlling current is oriented corresponding to a orientation of branch in which it is placed.

Substituting equations (6) up to (11) into equation (5) we obtain the matrix formulation of nodal equations for linear circuits having independent and/or dependent sources:

$$\mathbf{G}^{*(\mathrm{n})}\mathbf{U}^{(\mathrm{n})} = \mathbf{I}^{(\mathrm{n})}, \qquad (12)$$

where $\mathbf{G}^{*(\mathrm{n})}$ is a node conductance matrix of order $(N_\mathrm{u} - 1, N_\mathrm{u} - 1)$, $\mathbf{I}^{(\mathrm{n})}$ being a column matrix of short-circuiting currents produced by the current sources and voltage sources of order $(N_\mathrm{u} - 1, 1)$.

The obtained system of the nodal equations (12) must be treated for ideal current and voltage sources. An internal resistance of an ideal current source that is connected in the *k*th circuit branch is infinitely large and so its internal conductance is equal zero. An internal resistance of an ideal voltage source that is connected in the *k*th circuit branch is equal zero and so its internal conductance is infinitely large ($g_{kk} \to \infty$). For that reason it is necessary to arrange the system of nodal equation (12) as follows:

– if a voltage source is connected between non reference node p and reference node, a limit for $g_{kk} \to \infty$ of the left-hand side of equation written for nonreference node p must be found

$$\lim_{g_{kk}\to\infty}\left(\sum_{j=1}^{N_\mathrm{u}-1}\frac{g_{pj}^{*(\mathrm{n})}}{g_{kk}}u_j^{(\mathrm{n})}\right), \qquad (13)$$

as well as a limit for $g_{kk} \to \infty$ of right-hand side of equation written for nonreference node p

$$\lim_{g_{kk}\to\infty}\frac{i_p^{(\mathrm{n})}}{g_{kk}}, \qquad (14)$$

and the constraint equation in the form

$$u_p^{(\mathrm{n})} = \pm u_k^{(\mathrm{S})} \qquad (15)$$

will be obtained by this arrangement,

– if a voltage source is connected between a non reference node p and a non reference node q, a limit for $g_{kk} \to \infty$ of the both sides of equation written for nonreference node p must be found and the constraint equation in the form

$$u_p^{(\mathrm{n})} - u_q^{(\mathrm{n})} = \pm u_k^{(\mathrm{S})} \qquad (16)$$

will be obtained by this arrangement; but one more equation is needed, which will be provided by a nodal equation for supernode $p-q$ in such a way, that we add the equation for nonreference node p and the equation for non reference node q

$$\sum_{j=1}^{N_\mathrm{u}-1} \left(g_{pj}^{*(\mathrm{n})} + g_{qj}^{*(\mathrm{n})} \right) u_j^{(\mathrm{n})} = i_p^{(\mathrm{n})} + i_q^{(\mathrm{n})} . \qquad (17)$$

If any coefficient $g_{pj}^{*(\mathrm{n})}$ in the equation (17) includes a term $\pm g_{kk}$ (independent voltage source) or a term $\pm \rho_k g_{kk} g_{ll}$ (CCVS) or a term $\pm \alpha_k g_{kk}$ (VCVS), then a coefficient $g_{qj}^{*(\mathrm{n})}$ includes a term $\mp g_{kk}$ or a term $\mp \rho_k g_{kk} g_{ll}$ or a term $\mp \alpha_k g_{kk}$, and there is no any coefficient with term g_{kk} in the equation (17).

EXPERIMENTAL PART AND RESULTS OBTAINED. After assigning an input data (parameters of circuit elements, the incidence matrix and the reference node) a program generates a symbolic system of nodal equations. This system is subsequently solved and the obtained results are in symbolic and numeric form. In next part a solving of a circuit containing independent and dependent sources will be presented.

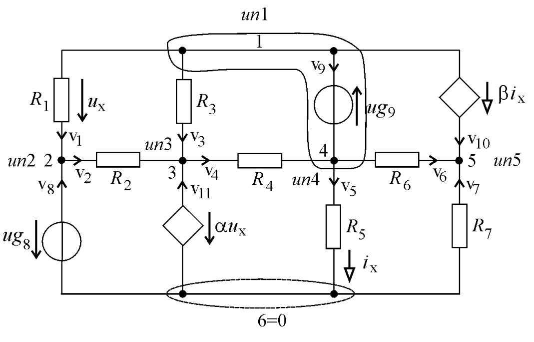

Figure 2 – Solved Circuit

Referring to Fig. 2, the program generates the system of equations (Fig. 3), which consists of three constraint equations and two nodal equations, one of them is for a non reference node 5 and the other one is for a supernode 1–4.

```
Constraint equations:
    un2 = ug8
    -alfa1*un1+alfa1*un2+un3 = 0
    -un1+un4 = ug9

Nodal equations for non reference nodes:
  node 5: (-beta1*g5-g6)*un4+(g6+g7)*un5 = 0

Nodal equations for supernodes:
  supernode 1-4: (g1+g3)*un1-g1*un2+(-g3-g4)*un3+(beta1*g5+g4+g5+g6)*un4-g6*un5 =
```

Figure 3 – Symbolic equations generated by program

For given values of circuit parameters: $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = 1\mathrm{k}\Omega$, $ug_8 = 12$ V, $ug_9 = 6$ V, $ig_{10} = \beta i_\mathrm{x} = 2\, i_\mathrm{x}$ A, $ug_{11} = \alpha\, u_\mathrm{x} = 2\, u_\mathrm{x}$ V, the program computes:

– the numeric values of nodal voltages (Fig. 4):

```
Nodal voltages - numeric values:
  un1 =-38.000000 V
  un2 = 12.000000 V
  un3 =-100.000000 V
  un4 =-32.000000 V
  un5 =-48.000000 V
```

Figure 4 – Nodal voltages

– the symbolic expressions of branch voltages in terms of nodal voltages and their numeric values (Fig. 5):

```
Branch voltages:
  uv1 = un1-un2 = -50.000000 V
  uv2 = un2-un3 = 112.000000 V
  uv3 = un1-un3 = 62.000000 V
  uv4 = un3-un4 = -68.000000 V
  uv5 = un4 = -32.000000 V
  uv6 = un4-un5 = 16.000000 V
  uv7 = -un5 = 48.000000 V
  uv8 = -un2 = -ug8 = -12.000000 V
  uv9 = un1-un4 = -ug9 = -6.000000 V
  uv10 = un1-un5 = 10.000000 V
  uv11 = -un3 = -2*uv1 = 100.000000 V
```

Figure 5 – Branch voltages

– the symbolic expressions of branch currents in terms of nodal voltages and their numeric values (Fig. 6):

```
Branch currents:

iv1 = (un1-un2)*g1 = -0.050000  A
iv2 = (un2-un3)*g2 =  0.112000  A
iv3 = (un1-un3)*g3 =  0.062000  A
iv4 = (un3-un4)*g4 = -0.068000  A
iv5 = un4*g5 = -0.032000  A
iv6 = (un4-un5)*g6 =  0.016000  A
iv7 = -un5*g7 =  0.048000  A
iv8 = -iv1+iv2 =  0.162000  A
iv9 = -iv1-iv3-iv10 =  0.052000  A
iv10 = 2*iv5 = -0.064000  A
iv11 = -iv2-iv3+iv4 = -0.242000  A
```

Figure 6 – Branch currents

CONCLUSIONS. The paper is aimed at matrix formulation of nodal analysis for linear electric circuits having independent and/or dependent sources. The program based on this method was created in MATLAB environment. The mentioned program generates symbolic expression of equations which are needed for nodal analysis of circuit, symbolic expressions and numeric values of branch voltages and branch currents in terms of nodal voltages. It is confirmed the fact that MATLAB is considered to provide a powerful and flexible environment also for programs.

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МОДЕЛИРОВАНИЕ ЭЛЕКТРИЧЕСКИХ ЦЕПЕЙ В СРЕДЕ MATLAB

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Рассматривается моделирование электрических систем, содержащих независимые и зависимые источники постоянного тока и/или напряжения. Для моделирования электрических цепей используется метод узловых потенциалов. Программа симуляции, основанная на использовании данного метода, разработана в среде MATLAB. Символьное представление уравнений связи для узлов напряжения и узловых уравнений для несвязанных узлов и «суперузлов» выполняется после установления принятых входных данных и узловых соотношений. Также могут быть рассчитаны символьное представление узловых напряжений, напряжений ветвей и токов ветвей, а также вычисления их числовых значений для заданных параметров элемента электрической цепи.

Ключевые слова: моделирование электрических цепей, линейные электрические цепи, узловой анализ, матрица событий, MATLAB.

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