UDC 621.313

MODELLING OF FIELD PROBLEMS IN MATLAB

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Modelling by using MATLAB Partial differential equation (PDE) Toolbox is proposed. This toolbox contains the three types of partial differential equations (elliptic, parabolic and hyperbolic equation) that are used as the mathematical models for a wide variety of phenomena in all branches of engineering and science. As examples of the field problems two types of problems were used. The first one is a problem in magnetostatics – the distribution of magnetic field in an element of magnetoelastic sensor. The second one is a problem of plane stress in structural mechanics - the distribution of stress field in an element of magnetoelastic sensor. The components of magnetic field as well as the components of stress field were carried out by a numerical solution of responding boundary–value problems.

Key words: partial differential equation of elliptic type, boundary-value problem, magnetic field distribution, stress field distribution.

Introduction. Computer modelling and simulation of various physical phenomena has become an essential tool for persons who work with complicated systems or who desire to understand the behaviour of such systems. The most common reasons for modelling and simulation are either a pure impossibility of experiment realization or enormous expenses for testing a real-world system. Modelling of field problems has a specific attribute because such problems can't be described by a few discrete variables, but they are better understandable if continuous functions of appropriate quantities over some region of space are used. As example of field model it can be a distribution of mechanical plane stress and static magnetic field in a magnetoelastic sensor element. For solving field problems it can be used professional packages as ANSYS [1], OPERA [2], MAGNET [3], etc., but also is useful MATLAB, which is a powerful, comprehensive, and easy-to-use environment for performing technical computations [4].

Problem statement. For determination of static magnetic field distribution and stress field distribution it is necessary to formulate an appropriate boundary-value problem. The boundary-value problem formulation requires defining the investigated domain with the boundary conditions and formulating the correct partial differential equation (PDE) or PDE system with correct coefficients [5-6].

Previous researches analysis. The determination of planar static magnetic field distribution in bounded domain is usually formulated as a boundary-value problem in terms of vector magnetic potential $\boldsymbol{A}$. The correct PDE is:

$$-\mathrm{div}\left(\frac{1}{\mu}\,\mathrm{grad}\,\boldsymbol{A}\right) = \boldsymbol{J}\ , \qquad (1)$$

where J being current density and μ being the magnetic permeability.

From mathematical point of view, the equation (1) represents PDE of elliptic type.

This PDE must be completed with the outer boundary conditions and the interface conditions along the subdomain borders between the regions of different material properties.

The outer boundary conditions are following ones:

for each point on the boundary must be specified a value of vector magnetic potential - Dirichlet boundary condition;

for each point on the boundary must be specified the derivative of vector magnetic potential by the outward normal vector - Neumann boundary condition;

for each point on the boundary must be specified a relation of vector magnetic potential to the derivative of vector magnetic potential by the outward normal vector - Newton boundary condition.

The interface conditions along the subdomain borders between the regions of different material properties are satisfied automatically and they do not require special treatment since the variation formulation of the boundary-value problem is used in PDE Toolbox.

The spatial distribution of the magnetic vector potential defines the spatial distribution of the magnetic flux density unambiguously. For the case of planar field the module B of magnetic flux density can be written in the form:

$$B = \sqrt{B_{\mathrm{x}}^2 + B_{\mathrm{y}}^2}\ , \qquad (2)$$

where B_{x} is the *x*-direction magnetic flux density, B_{x} being the *y*-direction flux density.

The determination of planar stress field distribution can be formulated as a boundary value problem too, but in terms of the displacement components. For plane stress, in the body made of the homogeneous isotropic material, equations for the balance of force are:

$$\frac{\partial \sigma_{\mathrm{x}}}{\partial x} + \frac{\partial \tau_{\mathrm{xy}}}{\partial y} + X = 0\ ; \qquad (3)$$

$$\frac{\partial \sigma_{\mathrm{y}}}{\partial y} + \frac{\partial \tau_{\mathrm{xy}}}{\partial x} + Y = 0\ , \qquad (4)$$

where σ_{x} is the *x*-direction stress, σ_{y} being the *y*-direction stress, τ_{xy} being the shear stress and *X*, *Y* being the volume forces.

The stress components ($\sigma_{\mathrm{z}}=0$) are closely related to the strain components and these relations are defined by Hooke law:

$$\begin{bmatrix}\sigma_{\mathrm{x}}\\ \sigma_{\mathrm{y}}\\ \tau_{\mathrm{xy}}\end{bmatrix}=\frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\begin{bmatrix}\varepsilon_{\mathrm{x}}\\ \varepsilon_{\mathrm{y}}\\ \gamma_{\mathrm{xy}}\end{bmatrix}, \qquad (5)$$

where E is Young's modulus, ν being Poisson's ratio, ε_{x} is the *x*-direction strain, ε_{y} being the *y*-direction strain, γ_{xy} being the shear strain.

The strain components can be expressed by the displacements:

$$\varepsilon_{\mathrm{x}}=\frac{\partial u}{\partial x},\quad \varepsilon_{\mathrm{y}}=\frac{\partial v}{\partial y},\quad \gamma_{\mathrm{xy}}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}, \qquad (6)$$

where u is the *x*-direction displacement and *v* being the *y*-direction displacement.

Combining the equations (3) – (6) and assuming that there are no volume forces, we arrived at the PDE system for balance of force in terms of the displacement components. The system takes form:

$$\begin{aligned}&\frac{E}{1-\nu^2}\frac{\partial^2 u}{\partial x^2}+\frac{E}{2(1+\nu)}\frac{\partial^2 u}{\partial y^2}+\\&+\nu\frac{E}{1-\nu^2}\frac{\partial^2 v}{\partial x\partial y}+\frac{E}{2(1+\nu)}\frac{\partial^2 v}{\partial y\partial x}=0\end{aligned}\ ; \qquad (7)$$

$$\begin{aligned}&\frac{E}{2(1+\nu)}\frac{\partial^2 v}{\partial x^2}+\frac{E}{1-\nu^2}\frac{\partial^2 v}{\partial y^2}+\\&+\frac{E}{2(1+\nu)}\frac{\partial^2 u}{\partial x\partial y}+\nu\frac{E}{1-\nu^2}\frac{\partial^2 u}{\partial y\partial x}=0\end{aligned}\ . \qquad (8)$$

From mathematical point of view, the equations (7) and (8) represent a system of two PDE of elliptic type.

This PDE system must be completed with following boundary conditions for each point on the boundary must be specified the value of the displacement:

$$u=g_1,\quad v=g_2, \qquad (9)$$

where g_1, g_2 being values of the displacement.

The boundary condition (9) is called Dirichlet boundary condition,

for each point on the boundary must be known the surface load (the derivative of the displacement by the outward normal vector):

$$\begin{aligned}&e_n\left(\begin{bmatrix}\frac{E}{1-\nu^2} & 0\\ 0 & \frac{E}{2(1+\nu)}\end{bmatrix}\nabla u\right)+\\&+e_n\left(\begin{bmatrix}0 & \frac{\nu E}{1-\nu^2}\\ \frac{E}{2(1+\nu)} & 0\end{bmatrix}\nabla v\right)=p_{\mathrm{x}}\end{aligned}\ ; \qquad (10)$$

$$\begin{aligned}&e_n\left(\begin{bmatrix}0 & \frac{E}{2(1+\nu)}\\ \frac{\nu E}{1-\nu^2} & 0\end{bmatrix}\nabla u\right)+\\&+e_n\left(\begin{bmatrix}\frac{E}{2(1+\nu)} & 0\\ 0 & \frac{E}{1-\nu^2}\end{bmatrix}\nabla v\right)=p_{\mathrm{y}}\end{aligned}\ , \qquad (11)$$

where e_n is the outward normal vector of the boundary, ∇ being Hamilton operator, p_{x} being the pressure in the direction of *x*-axis, p_{y} being the pressure in the *y*-direction.

The boundary conditions (10) and (11) are called Neumann boundary conditions.

Experimental part and obtained results. Both of these boundary-value problems were solved by professional code PDE Toolbox in an element of magnetoelastic sensor [7]. The element of magnetoelastic sensor (fig. 1) is made of ferromagnetic transformer sheet, thickness of which being 0.5 mm. It has squared shape with a circle hole in the middle. The radius of the hole is 1 mm.

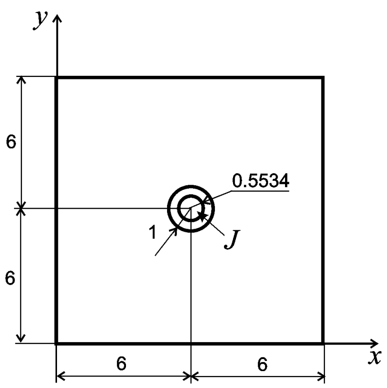


Figure 1 – An element of magnetoelastic sensor

In case that there is no force acting on sensor the boundary-value problem is nonlinear but isotropic one. In case that there exists force acting on the sensor, then the presented problem is nonlinear anisotropic problem.

The electromagnetic field in the element of magnetoelastic sensor was considered to be static although it is produced by a harmonic current of density *J* and frequency *f*. It was assumed that:

the displacement currents are small compared to the source current density and, therefore, they can be neglected on the right-hand side of the first Maxwell equation;

electrical conductivity of laminated sensor core is small, so that in the first approach the eddy current density can be neglected.

The static magnetic field was solved in the planar domain, consisting of three subdomains (fig. 2).

These subdomains are:

the copper current wire - in this subdomain $\mu=\mu_0$, J is the current density in conductors of the primary winding (J =13.512A/mm^2), and the equation (1) takes the following form:

$$\frac{\partial^2 \boldsymbol{A}}{\partial x^2}+\frac{\partial^2 \boldsymbol{A}}{\partial y^2}=-\mu \boldsymbol{J} ;$$

the air gap between the wire and ferromagnetic core - in this subdomain $\mu=\mu_0$, $J=0$ A/mm^2, and the equation (1) takes the following form:

$$\frac{\partial^2 \boldsymbol{A}}{\partial x^2}+\frac{\partial^2 \boldsymbol{A}}{\partial y^2}=0 ;$$

the ferromagnetic sensor core - in this subdomain $J=0$ A/mm^2, the permeability depends on the magnetic flux density $\mu=\mu(B)$ (in case that no pressure force is applied to the sensor) and the equation (1) takes the following form:

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial \boldsymbol{A}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial \boldsymbol{A}}{\partial y}\right)=-\boldsymbol{J} .$$

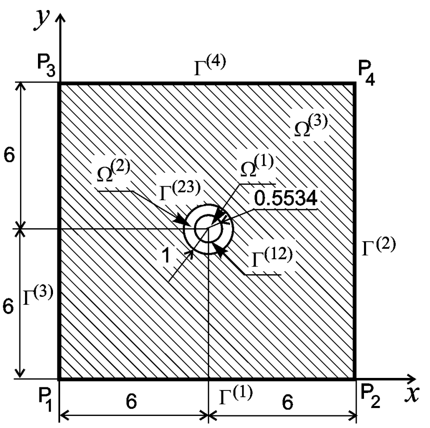


Figure 2 – The domain for determination of the magnetic field distribution

The magnetisation characteristic was measured in order to compute ferromagnetic material permeability. For the obtained set of data points (B,μ) the curve fitting by polynomial was done for frequency f = 400 Hz, because the permeability is also a function of frequency. The relevant measured points and corresponding approximating polynomial curves are depicted in fig. 3.

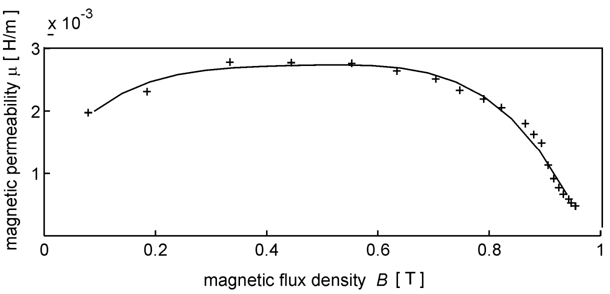


Figure 3 – The dependence of the permeability on the magnetic flux density $\mu=\mu(B)$

For solved problem are valid following outer boundary conditions:

at the boundary $\Gamma^{(1)}$, $\Gamma^{(4)}$ (fig. 2):

$$A=0,$$

because the field outside the sensor element is neglected;

at the boundary $\Gamma^{(2)}$, $\Gamma^{(3)}$ (fig. 2):

$$A=0,$$

the influence of neighbouring conductors at these boundaries is negated.

The interface conditions along the subdomain borders between the regions of different material properties are following ones:

at the subdomain border $\Gamma^{(5)}$ between $\Omega^{(3)}$ and $\Omega^{(2)}$ (fig. 2):

$$\frac{1}{\mu^{(3)}}\frac{\partial A^{(3)}}{\partial n}=\frac{1}{\mu^{(2)}}\frac{\partial A^{(2)}}{\partial n},$$

this condition arises from requirement that tangential components of vector magnetic potential must be equal at the subdomain border;

at the subdomain border $\Gamma^{(6)}$ between $\Omega^{(2)}$ a $\Omega^{(1)}$ (fig. 2):

$$\frac{\partial A^{(2)}}{\partial n}=\frac{\partial A^{(1)}}{\partial n}.$$

This is the requirement for the subdomain border air – nonmagnetic material.

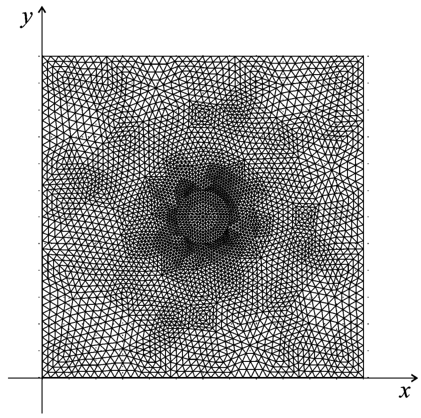


Figure 4 – The triangular mesh in the domain for determination of the magnetic field distribution

After solving a magnetic field problem, the obtained results at the nodal points of the given triangular mesh - consisting of 15777 nodes and 31232 triangles (fig. 4) were visualized. In fig. 4 the equipotential lines of the magnetic vector potential are plotted.

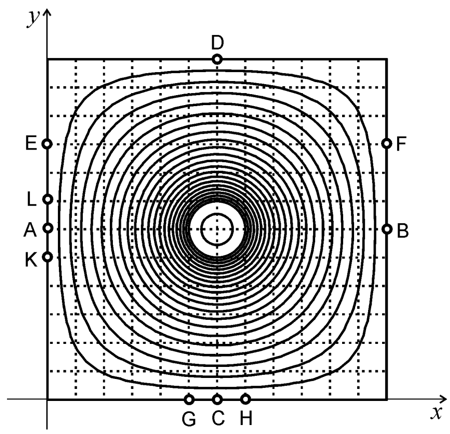


Figure 5 – The equipotential lines of the vector magnetic potential

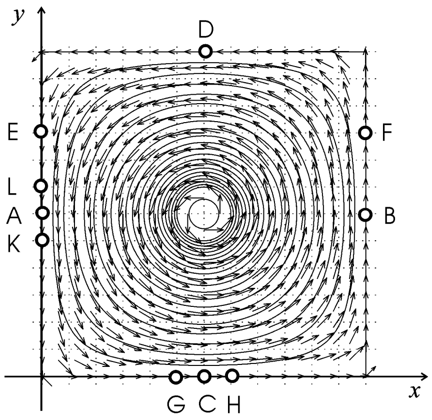


Figure 6 – The field of magnetic flux density vector

The obtained results can be visualised not only as the equipotential lines of the vector magnetic potential (fig. 5), but also as the field of the magnetic flux density vector (fig. 6).

The magnetic vector potential has no specific technical meaning. For that reason, the values of magnetic flux density over the rectangular grid (created with the step 0.01 mm in direction of x and y coordinates) were computed from the magnetic vector potential values on the triangular mesh.

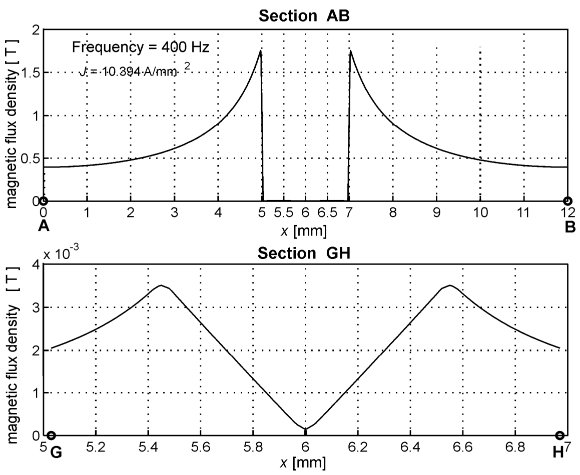


Figure 7 – The distribution of the magnetic flux density along the cross-section AB

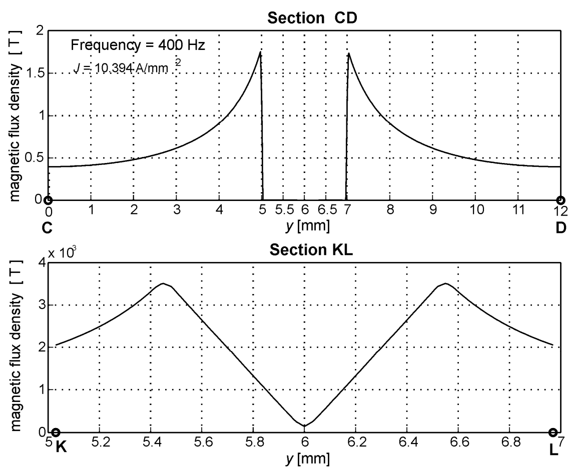


Figure 8 – The distribution of the magnetic flux density along the cross-section CD

The distribution of the magnetic flux density along the cross-sections AB, CD and EF in the element of magnetoelastic sensor is depicted in fig. 7 – 9 (current density in the primary winding is 13.512A/mm^2, its frequency being 400 Hz).

The stress field was solved in the same element, which was exposed to a constant continuous pressure stress in the direction of x-axis. The element is made of ferromagnetic transformer sheet, whose parameters are: $E = 1.86.10^5$ MN/m^2, $\nu = 0.3$.

The domain, in which the stress field will be solved, is surrounded by five boundaries $\Gamma^{(1)}_{u,v}$, $\Gamma^{(2)}_{u,v}$, $\Gamma^{(3)}_{u,v}$, $\Gamma^{(4)}_{u,v}$ and $\Gamma^{(5)}_{u,v}$ (fig. 10).

The boundary conditions for the displacement components are following:

the boundary $\Gamma^{(2)}_{u,v}$ (fig. 10) is subjected to a pressure $p = -p_X e_X$:

$$\frac{E}{1-\nu^2}\frac{\partial u}{\partial x} + \frac{\nu E}{1-\nu^2}\frac{\partial v}{\partial y} = -p_X ;$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0 ,$$

there is no displacement on the boundary $\Gamma^{(3)}_{u,v}$ (fig. 10):

$$u = 0 ;$$

$$v = 0 ,$$

the remaining boundaries $\Gamma^{(1)}_{u,v}$, $\Gamma^{(4)}_{u,v}$ and $\Gamma^{(5)}_{u,v}$ are free (fig. 10):

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0 ;$$

$$\nu\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 .$$

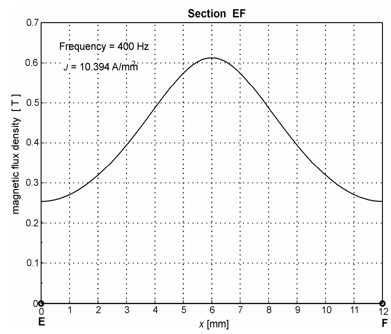


Figure 9 – The distribution of the magnetic flux density along the cross-section EF

The solution was made by using the adaptive mode option, which enables to refine the mesh in areas where the gradient of the solution is large in order to increase the accuracy of the solution [4]. The obtained results (for generated triangular mesh consisting of 42 112 nodes and 83 456 triangles) at the pressure force $p_X = 8.23$ MPa are depicted in fig. 12 – 14. In these figures are plotted the contour lines, e.g. the lines connected points of equal value of the depicted quantity in the solved domain.

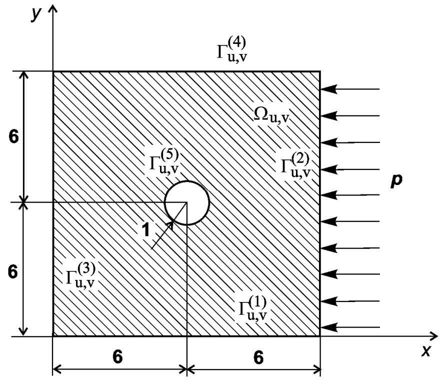


Figure 10 – The domain for determination of the stress field distribution

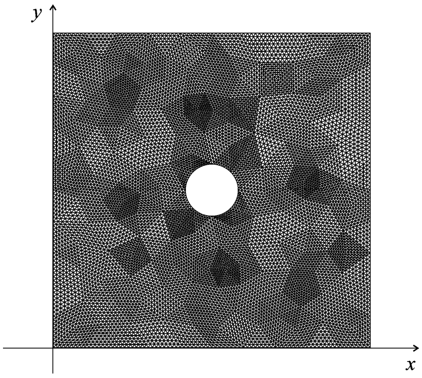
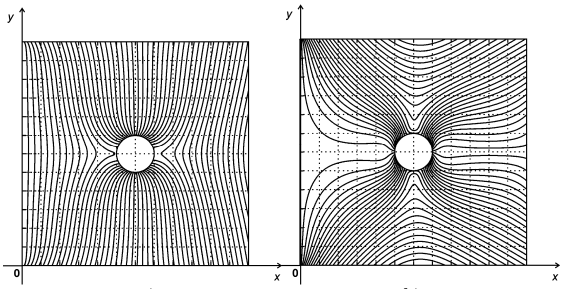


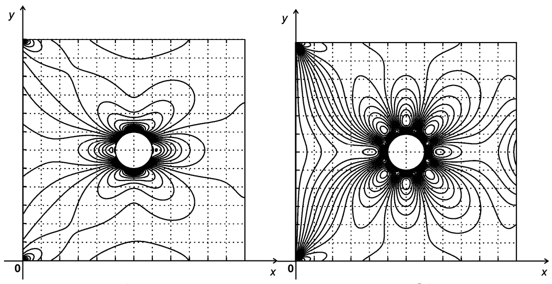
Figure 11 – The triangular mesh in the domain for determination of the stress field distribution

In fig. 12 are visualised the displacements $u(x, y)$ in the x–direction and $v(x, y)$ in the y–direction. The x–direction strain and the y–direction strain are depicted in fig. 13. The x–direction stress and the y–direction stress are shown in fig. 14.



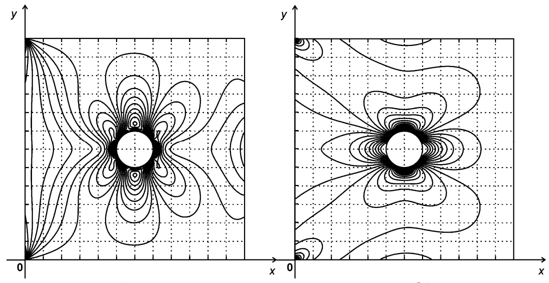
a) b)

Figure 12 – The displacement $u(x, y)$ in the x-direction (a) and $v(x, y)$ in the y-direction (b)



a) b)

Figure 13 – The strain ε_x in the x – direction (a) and ε_y in the y – direction (b)



a) b)

Figure 14 – The stress σ_x in the x-direction (a) and σ_y in the y-direction (b)

Conclusions. The paper is aimed at modelling field continuous problems. As example was used assignment of magnetic and stress field distribution in the element of magnetoelastic sensor. The distribution of these two fields was solved as the boundary-value problems in terms of the magnetic vector potential and displacement components by professional code PDE Toolbox in package MATLAB. Using of the toolbox confirmed the fact that this toolbox provides a powerful and flexible environment not only for solution of elliptic PDE, but it also handles solution of system of partial differential equations PDE.

Acknowledgement. The paper has been prepared under the support of Slovak grant project KEGA 003-003TUKE-4/2010.

We support research activities in Slovakia / Project is co-financed from EU funds. This paper was developed within the Project "Centre of Excellence of the Integrated Research & Exploitation the Advanced Materials and Technologies in the Automotive Electronics ", ITMS 26220120055.

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Стаття надійшла 06.06.2011 р.
Рекомендовано до друку к.т.н., доц. Гладирем А.І.

МОДЕЛИРОВАНИЕ ПРОБЛЕМ ПОЛЯ В MATLAB

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Предлагается моделирование уравнений частных производных с использованием пакета MATLAB. Этот программный продукт содержит три типа уравнений частных производных (эллиптические, параболические и гиперболические уравнения), которые используются в качестве математических моделей во всех отраслях техники и науки. В качестве примеров полевых задач были использованы два типа проблем. Первая проблема в магнитостатике – распределение магнитного поля в элементе магнитоупругих датчиков. Вторая проблема напряжений на плоскости в структурной механике – распределение поля напряжений в элементе магнитоупругих датчиков. Составляющие магнитного поля, а также составляющие поля напряжений проводились с помощью численного решения краевых задач.

Ключевые слова: уравнения частных производных эллиптического типа, краевая задача, распределение магнитного поля, распределение поля напряжений.

МОДЕЛЮВАННЯ ПРОБЛЕМ ПОЛЯ В MATLAB

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Пропонується моделювання рівнянь часткових похідних з використанням пакету MATLAB. Цей програмний продукт містить три типи рівнянь часткових похідних (еліптичні, параболічні та гіперболічні рівняння), які використовуються в якості математичних моделей у всіх галузях техніки і науки. Як приклади польових завдань були використані два типи проблем. Перша проблема в магнітостатиці – розподіл магнітного поля в елементі магнітопружних датчиків. Друга проблема напружень на площині у структурній механіці – розподіл поля напружень в елементі магнітопружних датчиків. Складові магнітного поля, а також складові поля напружень проводилися за допомогою чисельного рішення крайових задач.

Ключові слова: рівняння часткових похідних еліптичного типу, крайова задача, розподіл магнітного поля, розподіл поля напружень.