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# INSTANTANEOUS POWER SIGNAL ANALYSIS AND ITS COMPONENTS IN NONSINE CIRCUITS IN ELECTRICAL PROBLEM SOLVING

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A common approach to analysis and synthesis of the instantaneous power and voltage signal for non-sinusoidal circuits, which allows along with a quantitative evaluation make a qualitative analysis of the mechanism of formation of their spectra, as opposed to previously known, is considered. The problem of the spectrum transformation of a periodic non-sinusoidal signal in case of the coordinate system shift is considered. Approaches are actual in solving energy balance equations in problems of parametric identification of electric machines with regard to their non-linear nature and in synthesis of voltage signal for a given harmonic content of the instantaneous power in control quality of electrical energy conversion. The resulting mathematical relations are presented in conventional terms of signal theory and matrix algebra. On practice, this allows to synthesize high-speed computing procedures and it is important in design of quality control systems of energy conversion in real time.

**Key words:** instantaneous power, harmonic analysis, convolution, deconvolution.

# АНАЛІЗ СИГНАЛІВ МИТТЄВОЇ ПОТУЖНОСТІ ТА ЇЇ СКЛАДОВИХ У НЕСИНУСОЇДАЛЬНИХ ЛАНЦЮГАХ ПРИ ВИРІШЕННІ ЕЛЕКТРОТЕХНІЧНИХ ЗАВДАНЬ

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Розглянуто єдиний підхід щодо аналізу й синтезу сигналів миттєвої потужності й напруги для несинусоїдальних кіл, що дозволяє, на відміну від раніше відомих, поряд із кількісною оцінкою виконати якісний аналіз механізму формування їх спектрів. Розглянуто задачу перетворення спектру несинусоїдального періодичного сигналу в разі зсуву системи координат. Підходи актуальні при розв'язуванні рівнянь енергетичного балансу в задачах ідентифікації параметрів електричних машин з урахуванням їх нелінійного характеру й при синтезі сигналів напруги для заданого гармонічного складу миттєвої потужності в задачах управління якістю перетворення електричної енергії. Отримані математичні співвідношення представлено в загально прийнятих термінах теорії сигналів і матричної алгебри. На практиці це дозволить здійснити синтез швидкодіючих обчислювальних процедур і є важливим при проектуванні систем управління якістю перетворення енергії в реальному часі.

**Ключові слова:** миттєва потужність, гармонічний аналіз, згортка, зворотна згортка.

PROBLEM STATEMENT. The energy processes analysis in the power circuits of electrical drives (ED) [1–4] is the basis of their diagnostic systems, parameter identification and monitoring [5–7], where the primary source of information are signals of the instantaneous current values, voltage and power. A number of approaches to the analysis of energy processes in the ED [6, 8] require knowledge not only estimates of parameter values of harmonic components of the signal power, but also evaluation of the mechanism of their formation as a function of amplitudes of current and voltage signals (direct problem). A similar problem appears in a framework of the electric power metering conception in nonsinusoidal circuits, proposed in [9], for the correct calculation of active and reactive power components.

Electric drives consume and convert electrical energy into useful work. But energy is distorted in a process of its conversion. There are higher harmonics of voltage and current from power converters, non-sinusoidal currents from the presence of nonlinearities in power circuits, voltage disbalance of uneven load on phases, etc. All this is well known, and there are ways to reduce factors, mentioned above [10]. However, the electric motor, as a consumer of energy, is generally considered to be an electrically balanced electromechanical transducer without damage: currents in phases are symmetrical and sinusoidal, oscillations of the electromagnetic torque and speed are absent. Moreover, any deviations in the construction or motor parameters, that are acquired during repair or during long-term operation, make the engine essentially nonlinear system and lead to appearance of non-sinusoidal currents.

Mentioned above is also closely connected with the actual problem of quality management of electric energy transformation on a base of analyzing of instantaneous power, introduced by authors [7]. Authors solve the problem of a voltage signal synthesis at given signal waveforms of instantaneous power and current in electric power circuit (the inverse problem) for such management of an electromechanical system (EMS), where the energy converter, providing a given mode of the drive, not only transforms the energy, but also controls a quality of its transformation [11]. A similar approach is also used in the problem of the output formation voltage of power supply of systems for diagnostics of induction motors [12].

Mathematical aspects of calculation of instantaneous power components are considered, in particular, in works [13, 14]. Approach to the direct problem solution on a base of convolution of the spectra of current and voltage has been considered by authors in [15–17]. However, at the current moment development of the unified approach to both direct and inverse problems is very actual.

Work purpose is simplification the analysis and synthesis of signal spectra of voltage and instantaneous power in nonsinusoidal circuits in electromechanics's tasks by developing a common (general) approach to the parameters calculation in conventional terms of signal theory and matrix algebra.

In the first part of the article the direct problem is solved. The solution is based on a spectrum convolution of current and voltage. Scalar complex and real variants and their representation in terms of vector-matrix algebra are offered. In the second part the inverse problem of voltage signal synthesis on a base of given range of the instantaneous power is considered. Issues, that related to the spectrum transformation, caused by a shift of the coordinate system of studied signals, are discussed in the third part. The fourth part provides examples of electrical signals parameters calculation for induction motor (IM), which is connected to the thyristor voltage converter (TVC), with using relations, obtained in two previous parts.

EXPERIMENTAL PART AND RESULTS OBTAINED.

*1. Analysis and synthesis of a spectra of instantaneous power signal in nonsinusoidal circuits (direct problem)*

Let $u(t)$, $i(t)$ are signals of supply voltage, for example, in electric power circuit, $t \in [0,T]$, where $T$ is a signal period. Then the signal of instantaneous power can be written as $p(t) = i(t)u(t)$. Taking into account the periodic nature, $p(t)$ can be represented as Fourier series:

$$p(t) = \frac{P_0^a}{2} + \sum_{k=1}^{N} (P_k^a \cos k\omega t + P_k^b \sin k\omega t) \qquad (1)$$

or in its complex form:

$$p(t) = \sum_{k=-N}^{N} P_k e^{jk\omega t}, \qquad (2)$$

where $N$ is a number of the last significant harmonics in a signal, and

$$P = \frac{1}{2}\begin{bmatrix} P_{-N}^a \\ \vdots \\ P_{-k}^a \\ \vdots \\ P_{-1}^a \\ P_0^a \\ P_1^a \\ \vdots \\ P_k^a \\ \vdots \\ P_N^a \end{bmatrix} - j\frac{1}{2}\begin{bmatrix} -P_{-N}^b \\ \vdots \\ -P_{-k}^b \\ \vdots \\ P_{-1}^b \\ 0 \\ P_1^b \\ \vdots \\ P_k^b \\ \vdots \\ P_N^b \end{bmatrix} = \frac{1}{2}P^a - j\frac{1}{2}P^b. \qquad (3)$$

Spectra of signals $u(t)$, $i(t)$ are represented similarly. As noted above, the direct problem solution involves the assessment of the formation mechanism of the instantaneous power spectrum in relevant functional relations $P_k^a = f(I_k^a, U_k^a, I_k^b, U_k^b)$, $P_k^b = f(I_k^a, U_k^a, I_k^b, U_k^b)$ or $P_k = f(I_k, U_k)$.

For solve the task it is enough to use the convolution theorem, well-known from the signals theory, the essence of which is that the spectrum of the product of two signals is the convolution of their spectra: $P = I * U$. Then, taking into account the odd symmetry and the discrete nature of the spectra, integrated solution to the problem can be written as the following scalar ratio:

$$P_k = \sum_{\substack{i=0 \\ k-i\ge 0}}^{N} I_k U_{k-i}^* + \sum_{\substack{i=0 \\ k-i<0}}^{N} I_k U_{k-i}. \qquad (4)$$

Here the symbol "*" denotes the complex conjugate. Considering bilinear convolution:

$$P = I * U = \frac{1}{2}\left(I^a - jI^b\right) * \frac{1}{2}\left(U^a - jU^b\right) =$$

$$= \underbrace{\frac{1}{4}\left(I^a * U^a - I^b * U^b\right)}_{\mathrm{Re}(P)} - j\underbrace{\frac{1}{4}\left(I^a * U^b + I^b * U^a\right)}_{\mathrm{Im}(P)}.$$

$\Rightarrow$

$$P^a = 2\,\mathrm{Re}\{P\} = \frac{1}{2}\left(I^a * U^a - I^b * U^b\right); \qquad (5)$$

$$P^b = -2\,\mathrm{Im}\{P\} = \frac{1}{2}\left(I^a * U^b + I^b * U^a\right). \qquad (6)$$

Representing (5) and (6) as in (4) we obtain, respectively, amplitudes of cosine and sine components of signal's harmonics of instantaneous power as the following bilinear form:

$$P_k^a = \frac{1}{2}[\sum_{\substack{i=0 \\ k-i\ge 0}}^{N} I_i^a U_{k-i}^a + \sum_{\substack{i=0 \\ k-i<0}}^{N} I_i^a U_{i-k}^a - \sum_{\substack{i=0 \\ k-i\ge 0}}^{N} I_i^b U_{k-i}^b + \sum_{\substack{i=0 \\ k-i<0}}^{N} I_i^b U_{i-k}^b], k = \overline{0,2N}; \qquad (7)$$

$$P_k^b = \frac{1}{2}[\sum_{\substack{i=0 \\ k-i\ge 0}}^{N} I_i^b U_{k-i}^a + \sum_{\substack{i=0 \\ k-i<0}}^{N} I_i^b U_{i-k}^a + \sum_{\substack{i=0 \\ k-i\ge 0}}^{N} I_i^a U_{k-i}^b - \sum_{\substack{ik=0 \\ k-i<0}}^{N} I_i^a U_{i-k}^b], k = \overline{1,2N}. \qquad (8)$$

For compact representation and easy usage, in particular, to solve the inverse problem (see the next paragraph), (7) and (8) are expedient be presented in a matrix form

Let $E = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}$ is the identity matrix of dimension $N \times N$, and $J = \begin{pmatrix} 0 & \dots & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \vdots & \iddots & \vdots & \vdots \\ 1 & \dots & 0 & 0 \end{pmatrix}$ is the reflection matrix of the same dimension. We introduce the shift matrix dimension $N \times N$ similar to the [18, 19]:

$H = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}$. Note that $H^0 = E$, and

$H^2 = \begin{pmatrix} 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}$ etc. That is, during the exponentiation of the matrix $H$ to the power $n$ of the unit diagonal shifts to $n$ position to the right and up. Note also that

$$JU = \left(U_N \dots U_k \dots U_1 \dots 0 \dots - U_{-1} \dots - U_{-k} \dots - U_{-N}\right)^T.$$

Then a zero harmonic of the power can be represented as

$$P_0 = \frac{1}{4}\begin{bmatrix} I_{-N}^a + jI_{-N}^b \\ \vdots \\ I_{-k}^a + jI_{-k}^b \\ \vdots \\ I_{-1}^a + jI_{-1}^b \\ I_0^a \\ I_1^a - jI_1^b \\ \vdots \\ I_k^a - jI_k^b \\ \vdots \\ I_N^a - jI_N^b \end{bmatrix}^T \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix} \times$$

$$\times \begin{pmatrix} 0 & \dots & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \vdots & \iddots & \vdots & \vdots \\ 1 & \dots & 0 & 0 \end{pmatrix} \begin{bmatrix} U_{-N}^a + jU_{-N}^b \\ \vdots \\ U_{-k}^a + jU_{-k}^b \\ \vdots \\ U_{-1}^a + jU_{-1}^b \\ U_0^a \\ U_1^a - jU_1^b \\ \vdots \\ U_k^a - jU_k^b \\ \vdots \\ U_N^a - jU_N^b \end{bmatrix}$$

. Or in short description: $P_0 = I^T EJU = I^T H^0 JU$ . The second harmonic of signal is

$$P_2 = \frac{1}{4}\begin{bmatrix} I_{-N}^a + jI_{-N}^b \\ \vdots \\ I_{-k}^a + jI_{-k}^b \\ \vdots \\ I_{-1}^a + jI_{-1}^b \\ I_0^a \\ I_1^a - jI_1^b \\ \vdots \\ I_k^a - jI_k^b \\ \vdots \\ I_N^a - jI_N^b \end{bmatrix}^T \begin{pmatrix} 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix} \times$$

$$\times \begin{pmatrix} 0 & \dots & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \vdots & \iddots & \vdots & \vdots \\ 1 & \dots & 0 & 0 \end{pmatrix} \begin{bmatrix} U_{-N}^a + jU_{-N}^b \\ \vdots \\ U_{-k}^a + jU_{-k}^b \\ \vdots \\ U_{-1}^a + jU_{-1}^b \\ U_0^a \\ U_1^a - jU_1^b \\ \vdots \\ U_k^a - jU_k^b \\ \vdots \\ U_N^a - jU_N^b \end{bmatrix}$$

. Or in short description: $P_2 = I^T H^2 JU$ etc. We obtain for $k$ -th harmonic by induction method:

$$P_k = I^T H^k JU \,;\, k = \overline{0{,}2N} . \quad (9)$$

Equation (9) gives a block vector-column of dimension and a scalar dimension $2N \times 1$ :

$$\underline{P} = I * U = \begin{bmatrix} I^T JU \\ I^T H^1 JU \\ I^T H^2 JU \\ \vdots \\ I^T H^k JU \\ \vdots \\ I^T H^{2N} JU \end{bmatrix} = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ \vdots \\ P_k \\ \vdots \\ P_{2N} \end{bmatrix}. \quad (10)$$

Similarly, based on relations (7) and (8) with the use of the mathematical induction method, we can show that the amplitude of the cosine and sine component of the $k$ - th harmonic signal of instantaneous power expressed, respectively, by following relations:

$$P_k^a = \frac{1}{2}\left(I^{a^T} H^k JU^a - I^{b^T} H^k JU^b\right),\ k = \overline{0{,}2N} \,; \quad (11)$$

$$P_k^b = \frac{1}{2}\left(I^{a^T} H^k JU^b + I^{b^T} H^k JU^a\right),\ k = \overline{1{,}2N} \,, \quad (12)$$

where $N$ is a number of the last harmonic in current and voltage signals.

Relations (11), (12) generate block vector-columns of dimension and a scalar dimension $2N \times 1$ :

$$\underline{P^a} = \frac{1}{2}\begin{bmatrix} I^{a^T} JU^a - I^{b^T} JU^b \\ I^{a^T} HJU^a - I^{b^T} HJU^b \\ I^{a^T} H^2 JU^a - I^{b^T} H^2 JU^b \\ \vdots \\ I^{a^T} H^k JU^a - I^{b^T} H^k JU^b \\ \vdots \\ I^{a^T} H^{2N} JU^a - I^{b^T} H^{2N} JU^b \end{bmatrix} = \begin{bmatrix} P_0^a \\ P_1^a \\ P_2^a \\ \vdots \\ P_k^a \\ \vdots \\ P_{2N}^a \end{bmatrix}; \quad (13)$$

$$\underline{P^b} = \frac{1}{2}\begin{bmatrix} 0 \\ I^{a^T} JU^b + I^{b^T} JU^a \\ I^{a^T} H^2 JU^b + I^{b^T} H^2 JU^a \\ \vdots \\ I^{a^T} H^k JU^b + I^{b^T} H^k JU^a \\ \vdots \\ I^{a^T} H^{2N} JU^b + I^{b^T} H^{2N} JU^a \end{bmatrix} = \begin{bmatrix} 0 \\ P_1^b \\ P_2^b \\ \vdots \\ P_k^b \\ \vdots \\ P_{2N}^b \end{bmatrix}. \quad (14)$$

The first element of the vector (13) is a bilinear form of the constant component, produced as a result of multiplication only cosine or only sine of the current and voltage with the same frequency. The rest of elements of vectors (13) and (14) are canonical and noncanonical components. Canonical components of the instantaneous power are represented by bilinear forms, which are formed by the same frequencies of current and voltage harmonics. Noncanonical components of the instantaneous power are represented by bilinear forms, that created as a result of the multiplication of different frequencies of current and voltage components [3, 9] (see the example in section 4).

Thus, depending on the nature of solving problem, one can use either the complex representation of the power spectrum (9), or in terms of real cosine and sine amplitudes (13) and (14). Computationally, the last is advisable for cases of even or odd symmetry of signals as their sine or cosine components, respectively, are equal to zero.

Solving of the inverse problem (finding the voltage spectrum from the known spectra of the current and the instantaneous power) is also can be done on a base of the deconvolution of (9) or (13), (14), but from the algorithmic and computational point of view presents different tasks. The next section presents results of complex solution of the inverse problem based on (9).

*2. Analysis and synthesis of the voltage signal spectra in non-sinusoidal circuits (the inverse problem)*

A formal problem definition of synthesis of the voltage signal, the form of which is determined by signals $p(t)$ and $i(t)$ in power circuit of electric drive, can be written the next way. Let $i(t)$, $p(t)$ and its spectra $I$, $P$ are given (in terms of (3)). It's need to perform the synthesis of the voltage signal $u(t)$ and to find its spectrum $U$, i. e.

$$u(t) = \frac{p(t)}{i(t)} \leftrightarrow U_k \,;\; i(t) \neq 0 \,;\; k = \overline{0, N}\,. \quad (15)$$

The classical approach to the problem assumes the implementation of spectra $I$, $P$ deconvolution. Applying to both sides of (15) the Fourier transform operator $F[...]$ and using in the right side the inverse Fourier transform operator $F^{-1}[...]$, we obtain [18]:

$$U_k = F\left[\frac{F^{-1}[P_k]}{F^{-1}[I_k]}\right],\; k = 0, \pm 1, ..., \pm N\,. \quad (16)$$

Practically $F[...]$, $F^{-1}[...]$ are realized by using the fast Fourier transform algorithm. These is effectively from a computational point of view. However, the expression (16) is weakly (poorly) suitable for analysis.

Denote for compactness $I_{-k} = \dfrac{I_k^a + jI_k^b}{2}$ и $I_k^* = \dfrac{I_k^a - jI_k^b}{2}$ ), $k = \overline{0, N}$.

Expression $I^T H^k J$ in (9), where $k = \overline{0, N}$, is the block vector-column $\begin{bmatrix} I^T J \\ I^T H^1 J \\ I^T H^2 J \\ \vdots \\ I^T H^k J \\ \vdots \\ I^T H^{2N} J \end{bmatrix}$ of $2N \times 1$ dimension, that generate the Hankel triangular matrix $I_\Delta$ of dimension $2N \times 2N$:

$$I_\Delta = \begin{pmatrix} I_N^* & \dots & I_k^* & \dots & I_1^* & I_0 & I_{-1} & \dots & I_{-k} & \dots & I_{-N} \\ \vdots & I_k^* & \dots & I_1^* & I_0 & I_{-1} & \dots & I_{-k} & \dots & I_{-N} & 0 \\ I_k^* & \dots & I_1^* & I_0 & I_{-1} & \dots & I_{-k} & \dots & I-_N & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ I_1^* & I_0 & I_{-1} & \dots & I_{-k} & \dots & I-_N & 0 & 0 & 0 & 0 \\ I_0 & I_{-1} & \dots & I_k & \dots & I_{-N} & 0 & 0 & 0 & 0 & 0 \\ I_{-1} & \dots & I_{-k} & \dots & I_{-N} & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ I_{-k} & \vdots & I-_N & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ I_{-N} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, k = \overline{0, N}\,. \quad (17)$$

Then from (9) follows that

$$I_\Delta U = P\,. \quad (18)$$

This expression is a system of linear equations for $U$, which has an unique solution in case when

$$\det(I_\Delta) \neq 0\,. \quad (19)$$

Vector of complex amplitudes of voltage signal harmonics is uniquely expressed by the following relation:

$$U = I_\Delta^{-1} P\,, \quad (20)$$

where $I_\Delta$ is the Hankel triangular matrix of $2N \times 2N$ dimension of complex amplitudes of current signal in case when $\det(I_\Delta) \neq 0$.

It should be noted that $I^T H^k$ generates Toeplitz matrix and solution (20) can be reduced to finding an inverse vector of voltage harmonics $JU$.

Thus, the inverse problem solution reduces to inversion of Hankel or Toeplitz matrices, computational aspects of which are well known [18, 19]. Moreover, this approach allows a detailed analysis of the problem solution structure.

*3. Coordinate transformation*

The choice of a coordinate system and, consequently, invariants are important, according to vectors of components of current, voltage and instantaneous power harmonics as input parameters of equations for parameters estimation of circuits or quality of energy conversion [7]. For example, values of cosine and sine harmonic amplitudes, as well as values of their phases, are not invariant, while the modulus of the complex amplitude is invariant relatively to the coordinate system transformation, in which signals are considered.

At first it is natural to assume that the using of amplitudes of cosine and sine components of the current, voltage or instantaneous power in different reference frames as coefficients in estimation equations of equivalent circuit parameters, for example in diagnostic tasks of electromechanical systems, can lead to different values of errors calculations even at full absence of measurement errors.

On the other hand, it is clear that in real conditions a metrological aspect, which is important too, is added to a computational aspect. There will be a different estimation error of signals amplitudes for the same harmonics, that really present in signals, during selection of different starting points in the coordinate system in the process of harmonic analysis [20, 21]. One of special cases is considered in [21]. However, the development of a common (general) approach to the consideration of the coordinates transformation mechanism with respect to mentioned above signals with regard to issues, discussed in [22], is actual.

It means only a choice of a starting point in a harmonic analysis process according to the coordinate system transformation with respect to problems, discussed in [19]. As known, in a time domain it is equivalent to a signal shift in time by the amount of $\mp\tau$ in case of delay or advance of a signal, respectively (Fig. 1). In a spectral range it is equivalent to a rotation vector of the complex amplitude of harmonics to the corner $\mp\omega_0\tau$, where $\omega_0$ is a frequency of a fundamental (main) signal, "-" corresponds to direction of rotation by counterclockwise, and "+" corresponds to direction of rotation by clockwise.

In this case $u(t)=U_1\sin(\omega_0 t)$ and, therefore, $u(t\mp\tau)=U_1\sin(\omega_0(t\mp\tau))=U_1\sin(\omega_0 t\mp\omega_0\tau)$.

In this regard, the answer to the question how the spectrum of the initial polyharmonic (non-sinusoidal) signal will be changed in case of a shift in time domain on the value $\mp\tau$, is actual.

In other words, if $u(t)\leftrightarrow U_k$, $k=\overline{0,N}$, then $u(t\mp\tau)\leftrightarrow U'_k$?

From signal theory it is known [18] that $u(t-\tau)\leftrightarrow e^{-j\omega_0 k\tau}U_k$, $k=\overline{0,N}$. Then the unknown vector harmonics $U'$ can be written as

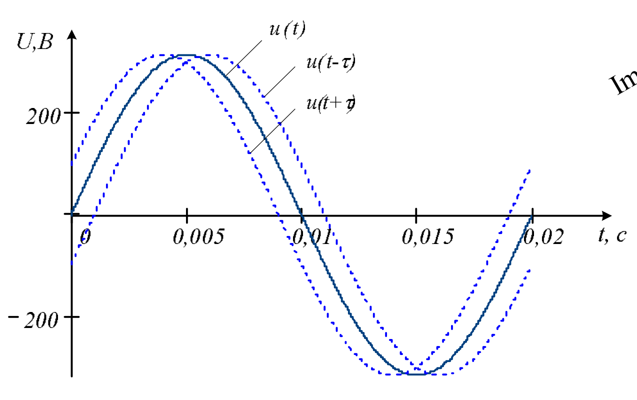
$$U'=\begin{bmatrix} U_0 \\ e^{\mp j\omega_0\tau}U_1 \\ e^{\mp j\omega_0 2\tau}U_2 \\ \vdots \\ e^{\mp j\omega_0 k\tau}U_k \\ \vdots \\ e^{\mp j\omega_0 N\tau}U_N \end{bmatrix}. \qquad (21)$$

Lets introduce an orthogonal matrix of complex sinusoids. Than expression (21) can be written as follows:

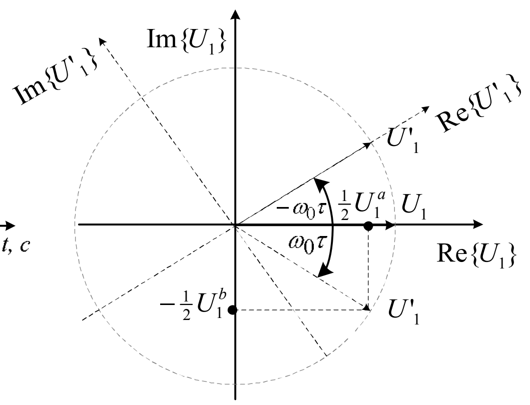
$U'=$

$$\begin{bmatrix} U'_0 \\ U'_1 \\ U'_2 \\ \vdots \\ U'_k \\ \vdots \\ U'_N \end{bmatrix}=\begin{bmatrix} 1 & 0 & \vdots & 0 & \vdots & 0 \\ 0 & e^{\mp j\omega_0\tau} & \vdots & 0 & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & e^{\mp j\omega_0 k\tau} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \vdots & 0 & 0 & e^{\mp j\omega_0 N\tau} \end{bmatrix}\begin{bmatrix} U_0 \\ U_1 \\ U_2 \\ \vdots \\ U_k \\ \vdots \\ U_N \end{bmatrix}, \qquad (22)$$

where "–" corresponds to a signal shift to the right on a value $\tau$, and the "+" corresponds to a signal shift to the left on the same value.




*а*)




*б*)

Figure 1 – Explains signal shift in time by example of the first harmonic of the voltage: a) in a time area; b) in a complex area

It is known that the value of $|U_k'| = \sqrt{U_k^{a\,2} + U_k^{b\,2}}$ of each component of the vector $U'$ is invariant to a shift, and the phase $\varphi_k = arctg\left(\frac{U_k^b}{U_k^a}\right)$ is not invariant, for $\forall k = \overline{1, N}$.

We obtain the scalar version of relations, similarly to (22).

Let $u(t) \leftrightarrow U_k = \frac{1}{2}U_k^a - j\frac{1}{2}U_k^b$, $k = \overline{0, N}$, than $u(t \mp \tau) \leftrightarrow U'_k = \frac{1}{2}U'^a_k - j\frac{1}{2}U'^b_k$. We introduce the vector equivalence $\frac{1}{2}U_k^a - j\frac{1}{2}U_k^b \sim \begin{bmatrix} U_k^a \\ U_k^b \end{bmatrix}$, $\forall k = \overline{0, N}$. We use a matrix of a linear transformation in Cartesian coordinates [19] for representation of cosine and sine component conversion:

$$\begin{bmatrix} U'^a_k \\ U'^b_k \end{bmatrix} = \begin{bmatrix} \cos(\omega_0 k\tau) & \mp\sin(\omega_0 k\tau) \\ \pm\sin(\omega_0 k\tau) & \cos(\omega_0 k\tau) \end{bmatrix} \begin{bmatrix} U_k^a \\ U_k^b \end{bmatrix}; \quad (23)$$

$\forall k = \overline{0, N}$, where «–» corresponds to a shift to the right on the value of signal $\tau$, a «+» corresponds to shift to the left on the same value. Generalizing the relation (23) to the case of $N$ harmonics, we obtain a block vector-column:

$$\begin{bmatrix} \begin{bmatrix} U'^a_0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} U'^a_1 \\ U'^b_1 \end{bmatrix} \\ \begin{bmatrix} U'^a_2 \\ U'^b_2 \end{bmatrix} \\ \vdots \\ \begin{bmatrix} U'^a_k \\ U'^b_k \end{bmatrix} \\ \vdots \\ \begin{bmatrix} U'^a_N \\ U'^b_N \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} U_0^a \\ 0 \end{bmatrix} \\ \begin{bmatrix} \cos(\omega_0\tau) & \mp\sin(\omega_0\tau) \\ \pm\sin(\omega_0\tau) & \cos(\omega_0\tau) \end{bmatrix} \begin{bmatrix} U_1^a \\ U_1^b \end{bmatrix} \\ \begin{bmatrix} \cos(\omega_0 2\tau) & \mp\sin(\omega_0 2\tau) \\ \pm\sin(\omega_0 2\tau) & \cos(\omega_0 2\tau) \end{bmatrix} \begin{bmatrix} U_2^a \\ U_2^b \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \cos(\omega_0 k\tau) & \mp\sin(\omega_0 k\tau) \\ \pm\sin(\omega_0 k\tau) & \cos(\omega_0 k\tau) \end{bmatrix} \begin{bmatrix} U_k^a \\ U_k^b \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \cos(\omega_0 N\tau) & \mp\sin(\omega_0 N\tau) \\ \pm\sin(\omega_0 N\tau) & \cos(\omega_0 N\tau) \end{bmatrix} \begin{bmatrix} U_N^a \\ U_N^b \end{bmatrix} \end{bmatrix} . (24)$$

We introduce an orthogonal block matrix of the linear transformation in Cartesian coordinates [19]. Than (24) can be written as:

$$\begin{bmatrix} \begin{bmatrix} U'^a_0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} U'^a_1 \\ U'^b_1 \end{bmatrix} \\ \begin{bmatrix} U'^a_2 \\ U'^b_2 \end{bmatrix} \\ \vdots \\ \begin{bmatrix} U'^a_k \\ U'^b_k \end{bmatrix} \\ \vdots \\ \begin{bmatrix} U'^a_N \\ U'^b_N \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & 0 & \vdots & 0 & \vdots & 0 \\ 0 & \begin{bmatrix} \cos(\omega_0\tau) & \mp\sin(\omega_0\tau) \\ \pm\sin(\omega_0\tau) & \cos(\omega_0\tau) \end{bmatrix} & \vdots & 0 & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \begin{bmatrix} \cos(\omega_0 k\tau) & \mp\sin(\omega_0 k\tau) \\ \pm\sin(\omega_0 k\tau) & \cos(\omega_0 k\tau) \end{bmatrix} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \vdots & 0 & 0 & \begin{bmatrix} \cos(\omega_0 N\tau) & \mp\sin(\omega_0 N\tau) \\ \pm\sin(\omega_0 N\tau) & \cos(\omega_0 N\tau) \end{bmatrix} \end{bmatrix} \times \begin{bmatrix} \begin{bmatrix} U_0^a \\ 0 \end{bmatrix} \\ \begin{bmatrix} U_1^a \\ U_1^b \end{bmatrix} \\ \begin{bmatrix} U_2^a \\ U_2^b \end{bmatrix} \\ \vdots \\ \begin{bmatrix} U_k^a \\ U_k^b \end{bmatrix} \\ \vdots \\ \begin{bmatrix} U_N^a \\ U_N^b \end{bmatrix} \end{bmatrix} . (25)$$

Obtained relations allow to present the spectrum of the measured signal in the form, harmonics' parameters of which are satisfy given requirements. For example, in several methods [9] the correct calculation of $P_k^a$ and $P_k^b$ as bilinear forms of the voltage amplitudes and current in accordance with methods, proposed in [22], suggests that the $U_1^b = 0$, or, in other words, requires zero phase of the first harmonic: $\varphi_1^U = 0$, if the voltage signal is represented as $u(t) = \sum_{k=0}^{\infty} 2|U_k|\cos\left(k\omega t - \varphi_k^U\right)$. In general, for the implementation of $u(t)$, obtained in the measurement process, $U_1^a \neq 0$, $U_1^b \neq 0$. Consider the problem of shifting the signal $u(t)$, which leads to the fulfillment of the condition:

$$U_1^a \neq 0; \; U_1^b = 0. \quad (26)$$

From relation (23) follows that

$$\begin{bmatrix} U'^a_1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\omega_0\tau) & \mp\sin(\omega_0\tau) \\ \pm\sin(\omega_0\tau) & \cos(\omega_0\tau) \end{bmatrix} \begin{bmatrix} U_1^a \\ U_1^b \end{bmatrix}, \quad (27)$$

then

$$U_1^a \sin(\omega_0\tau) + U_1^b \cos(\omega_0\tau) = 0. \quad (28)$$

Solution (28) has the form:

$$\tau = \frac{arctg\left(-\frac{U_1^b}{U_1^a}\right)}{\omega_0}; \; \omega_0 = 2\pi f; \; f = T^{-1}. \quad (29)$$

For (26) with a negative value of the parameter $\tau$ voltage signal must be shifted to the left on a value $\tau$, and with a positive value it must be shifted to the right.

Thus, substituting (29) into (22)–(24) or (25), we will obtain the spectrum of the shifted voltage signal. Calculations are similar for current and instantaneous power signals.

Solution of a similar problem for the $k$-th harmonic will be the following:

$$\tau = \frac{arctg\left(-\frac{U_k^b}{U_k^a}\right)}{k\omega_0}; \; \omega_0 = 2\pi f; \; f = T^{-1}. \quad (30)$$

In case of a signals shift in accordance with condition

$$U_1^a = 0; \; U_1^b \neq 0, \quad (31)$$

we obtain similarly the following solution:

$$\tau = \frac{arctg\left(\frac{U_k^a}{U_k^b}\right)}{k\omega_0}; \ \omega_0 = 2\pi f; \ f = T^{-1}. \qquad (32)$$

Obviously, in case of an even symmetry of a signal the shift will result in absence of sinus amplitudes and in case of odd symmetry the shift will results in lack of cosine amplitudes in the transformed signal spectrum:

$$\left[\begin{bmatrix} U'^a_0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ U'^b_1 \end{bmatrix} \begin{bmatrix} 0 \\ U'^b_2 \end{bmatrix} \cdots \begin{bmatrix} 0 \\ U'^b_k \end{bmatrix} \cdots \begin{bmatrix} 0 \\ U'^b_N \end{bmatrix}\right]^T;$$

$$\left[\begin{bmatrix} U'^a_0 \\ 0 \end{bmatrix} \begin{bmatrix} U'^a_1 \\ 0 \end{bmatrix} \begin{bmatrix} U'^a_2 \\ 0 \end{bmatrix} \cdots \begin{bmatrix} U'^a_k \\ 0 \end{bmatrix} \cdots \begin{bmatrix} U'^a_N \\ 0 \end{bmatrix}\right]^T.$$

.

Based on this, we can conclude that the bilinear form (4), (7) and (8) are not invariant to parameters and structure.

Taking into account that on practice we are dealing with discrete signals, we can write: $\tau = N_\tau \Delta t$, where $N_\tau$ is a number of discrete values that fit in the interval $\tau$; $\Delta t$ is a sampling interval in time. Then (29) takes the form:

$$N_\tau = \left[\frac{\tau}{\Delta t}\right] = \left[\frac{arctg\left(-\frac{U_1^b}{U_1^a}\right)}{\omega \Delta t}\right] = \left[\frac{arctg\left(-\frac{U_1^b}{U_1^a}\right)N}{2\pi}\right], \qquad (33)$$

where $[...]$ is an integer part of number; $N = \frac{T}{\Delta t}$ is a number of points per period. Similarly, we obtain the discrete version of the relations (30):

$$N_\tau = \left[\frac{\tau}{\Delta t}\right] = \left[\frac{arctg\left(-\frac{U_k^b}{U_k^a}\right)}{k\omega \Delta t}\right] = \left[\frac{arctg\left(-\frac{U_k^b}{U_k^a}\right)N}{2\pi k}\right]. \qquad (34)$$

Thus, if necessary, processing of signals $u(t \mp \tau)$, $i(t \mp \tau)$, $p(t \mp \tau)$ and a further harmonic analysis should be done, or transformation of obtained spectra according to relations (22)–(24) or (25) should be done.

*4. Examples of practical applications*

As an example of practical application of obtained above relations, we'll perform the calculation of amplitude values the instantaneous power signal components of IM, which operates in the regime of short circuit. Motor stator windings are connected to TVC with control angle $\alpha = 110^0$. IM parameters correspond to T-equivalent circuit: rated power of 75 kW, rated voltage of 380 V, the number of pole pairs 12, the resistance and inductance of the stator phase 0,015 Ohm and 0,015731 H, given resistance and inductance of the rotor phase 0,023 Ohm, 0,015719 H, magnetizing inductance 0,015 H. Current and voltage signals diagrams are obtained by solving a system of differential equations of IM, taking into account their calculation for IM models [23]. Waveforms of voltage and current on the repetition period of the process are shown in Fig. 2.

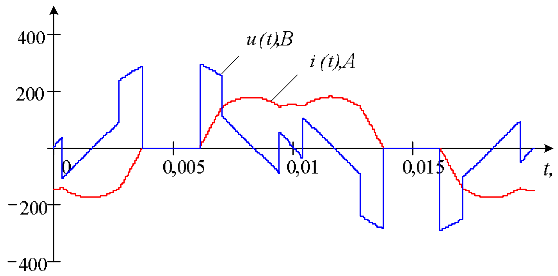



Figure 2 – Voltage and current signals

Appropriate signal of instantaneous power is shown in Fig. 2.

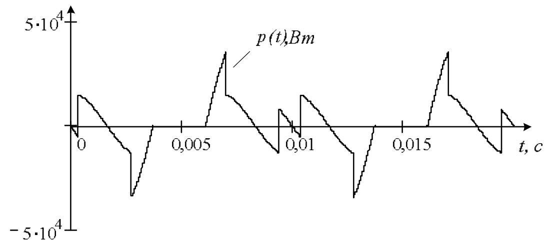



Figure 3 – Signal instantaneous power

Signal spectra are shown in Fig. 4

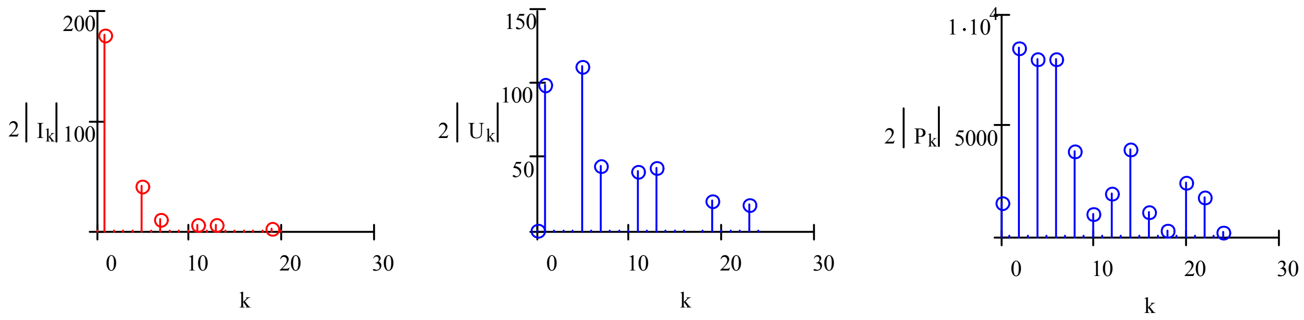



Figure 4 – Signal spectrum of current, voltage and instantaneous power

For simplicity, we consider only the first and fifth harmonics in voltage and current signals.

Based on conditions of the problem, and in according to (3), we obtain (35).

Vector of current complex amplitudes can be written similarly (36).

Then, substituting (35) and (36) respectively in (13) and (14), and after calculation, we get (37, 38).

$$U=\frac{1}{2}\begin{pmatrix} U_{-5}^{a}+jU_{-5}^{b}\\ 0\\ 0\\ 0\\ U_{-1}^{a}+jU_{-1}^{b}\\ 0\\ U_{1}^{a}-jU_{1}^{b}\\ 0\\ 0\\ 0\\ U_{5}^{a}-jU_{5}^{b}\end{pmatrix}=\frac{1}{2}\begin{pmatrix} U_{-5}^{a}\\ 0\\ 0\\ 0\\ U_{-1}^{a}\\ 0\\ U_{1}^{a}\\ 0\\ 0\\ 0\\ U_{5}^{a}\end{pmatrix}-j\frac{1}{2}\begin{pmatrix} jU_{-5}^{b}\\ 0\\ 0\\ 0\\ jU_{-1}^{b}\\ 0\\ -jU_{1}^{b}\\ 0\\ 0\\ 0\\ -jU_{5}^{b}\end{pmatrix}=\begin{pmatrix} -6{,}816-j55{,}798\\ 0\\ 0\\ 0\\ -2{,}889+j49{,}498\\ 0\\ -2{,}889-j49{,}498\\ 0\\ 0\\ 0\\ -6{,}816+j55{,}798\end{pmatrix}=\begin{pmatrix} -6{,}816\\ 0\\ 0\\ 0\\ -2{,}889\\ 0\\ -2{,}889\\ 0\\ 0\\ 0\\ -6{,}816\end{pmatrix}-j\begin{pmatrix} -55{,}798\\ 0\\ 0\\ 0\\ 49{,}498\\ 0\\ -49{,}498\\ 0\\ 0\\ 0\\ 55{,}798\end{pmatrix}. \qquad (35)$$

$$I=\frac{1}{2}\begin{pmatrix} I_{-5}^{a}+jI_{-5}^{b}\\ 0\\ 0\\ 0\\ I_{-1}^{a}+jI_{-1}^{b}\\ 0\\ I_{1}^{a}-jI_{1}^{b}\\ 0\\ 0\\ 0\\ I_{5}^{a}-jI_{5}^{b}\end{pmatrix}=\frac{1}{2}\begin{pmatrix} I_{-5}^{a}\\ 0\\ 0\\ 0\\ I_{-1}^{a}\\ 0\\ I_{1}^{a}\\ 0\\ 0\\ 0\\ I_{5}^{a}\end{pmatrix}-j\frac{1}{2}\begin{pmatrix} jI_{-5}^{b}\\ 0\\ 0\\ 0\\ jI_{-1}^{b}\\ 0\\ -jI_{1}^{b}\\ 0\\ 0\\ 0\\ -jI_{5}^{b}\end{pmatrix}=\begin{pmatrix} 20{,}004-j2{,}688\\ 0\\ 0\\ 0\\ -88{,}698+j2{,}433\\ 0\\ -88{,}698-j2{,}433\\ 0\\ 0\\ 0\\ 20{,}004+j2{,}688\end{pmatrix}=\begin{pmatrix} 20{,}004\\ 0\\ 0\\ 0\\ -88{,}98\\ 0\\ -88{,}98\\ 0\\ 0\\ 0\\ 20{,}004\end{pmatrix}-j\begin{pmatrix} -2{,}688\\ 0\\ 0\\ 0\\ 2{,}433\\ 0\\ -2{,}433\\ 0\\ 0\\ 0\\ 2{,}688\end{pmatrix}. \qquad (36)$$

$$\underline{P^{a}}=\frac{1}{2}\begin{pmatrix} I^{a^T}JU^{a}-I^{b^T}JU^{b}\\ I^{a^T}HJU^{a}-I^{b^T}HJU^{b}\\ I^{a^T}H^{2}JU^{a}-I^{b^T}H^{2}JU^{b}\\ I^{a^T}H^{3}JU^{a}-I^{b^T}H^{3}JU^{b}\\ I^{a^T}H^{4}JU^{a}-I^{b^T}H^{4}JU^{b}\\ I^{a^T}H^{5}JU^{a}-I^{b^T}H^{5}JU^{b}\\ I^{a^T}H^{6}JU^{a}-I^{b^T}H^{6}JU^{b}\\ I^{a^T}H^{7}JU^{a}-I^{b^T}H^{7}JU^{b}\\ I^{a^T}H^{8}JU^{a}-I^{b^T}H^{8}JU^{b}\\ I^{a^T}H^{9}JU^{a}-I^{b^T}H^{9}JU^{b}\\ I^{a^T}H^{10}JU^{a}-I^{b^T}H^{10}JU^{b}\end{pmatrix}=\begin{pmatrix} P_{0}^{a}\\ P_{1}^{a}\\ P_{2}^{a}\\ P_{3}^{a}\\ P_{4}^{a}\\ P_{5}^{a}\\ P_{6}^{a}\\ P_{7}^{a}\\ P_{8}^{a}\\ P_{9}^{a}\\ P_{10}^{a}\end{pmatrix}=\begin{pmatrix} I_{5}^{a}U_{5}^{a}+I_{1}^{a}U_{1}^{a}+I_{5}^{b}U_{5}^{b}+I_{1}^{b}U_{1}^{b}\\ 0\\ \frac{1}{2}I_{1}^{a}U_{1}^{a}-\frac{1}{2}I_{1}^{b}U_{1}^{b}\\ 0\\ \frac{1}{2}I_{1}^{a}U_{5}^{a}+\frac{1}{2}I_{5}^{a}U_{1}^{a}+\frac{1}{2}I_{1}^{b}U_{5}^{b}+\frac{1}{2}I_{5}^{b}U_{1}^{b}\\ 0\\ \frac{1}{2}I_{1}^{a}U_{5}^{a}+\frac{1}{2}I_{5}^{a}U_{1}^{a}-\frac{1}{2}I_{1}^{b}U_{5}^{b}-\frac{1}{2}I_{5}^{b}U_{1}^{b}\\ 0\\ 0\\ 0\\ \frac{1}{2}I_{5}^{a}U_{5}^{a}-\frac{1}{2}I_{5}^{b}U_{5}^{b}\end{pmatrix}; \qquad (37)$$

$$\underline{P^{b}}=\frac{1}{2}\begin{pmatrix} 0\\ I^{a^T}JU^{b}+I^{b^T}JU^{a}\\ I^{a^T}H^{2}JU^{b}+I^{b^T}H^{2}JU^{a}\\ I^{a^T}H^{3}JU^{b}+I^{b^T}H^{3}JU^{a}\\ I^{a^T}H^{4}JU^{b}+I^{b^T}H^{4}JU^{a}\\ I^{a^T}H^{5}JU^{b}+I^{b^T}H^{5}JU^{a}\\ I^{a^T}H^{6}JU^{b}+I^{b^T}H^{6}JU^{a}\\ I^{a^T}H^{7}JU^{b}+I^{b^T}H^{7}JU^{a}\\ I^{a^T}H^{8}JU^{b}+I^{b^T}H^{8}JU^{a}\\ I^{a^T}H^{9}JU^{b}+I^{b^T}H^{9}JU^{a}\\ I^{a^T}H^{10}JU^{b}+I^{b^T}H^{10}JU^{a}\end{pmatrix}=\begin{pmatrix} 0\\ P_{1}^{b}\\ P_{2}^{b}\\ P_{3}^{b}\\ P_{4}^{b}\\ P_{5}^{b}\\ P_{6}^{b}\\ P_{7}^{b}\\ P_{8}^{b}\\ P_{9}^{b}\\ P_{10}^{b}\end{pmatrix}=-\begin{pmatrix} 0\\ 0\\ \frac{1}{2}I_{1}^{b}U_{1}^{a}+\frac{1}{2}I_{1}^{a}U_{1}^{b}\\ 0\\ -\frac{1}{2}I_{1}^{b}U_{5}^{a}+\frac{1}{2}I_{5}^{b}U_{1}^{a}+\frac{1}{2}I_{1}^{a}U_{5}^{b}-\frac{1}{2}I_{5}^{a}U_{1}^{b}\\ 0\\ \frac{1}{2}I_{1}^{b}U_{5}^{a}+\frac{1}{2}I_{5}^{b}U_{1}^{a}-\frac{1}{2}I_{1}^{a}U_{5}^{b}+\frac{1}{2}I_{5}^{a}U_{1}^{b}\\ 0\\ 0\\ 0\\ \frac{1}{2}I_{5}^{b}U_{5}^{a}+\frac{1}{2}I_{5}^{a}U_{5}^{b}\end{pmatrix}. \qquad (38)$$

Bilinear forms (37) and (38) allow to perform analysis of the formation mechanism of harmonic components of the instantaneous power signal as functions of amplitude components of current and voltage signals. This is necessary, in particular, to solve systems of energy balance equations in parametric identification problems of equivalent circuit of electric machine [3, 4] and in problems of electrical energy metering [9]. Thus, from (37) and (38) it follows that the instantaneous power signal has a DC component (zero harmonic). Second and tenth components are canonical components, formed by multiplying the same frequency amplitudes of current and voltage. Fourth and sixth components are non-canonical components, formed by multiplying the amplitudes of different frequency of current and voltage.

Lets perform the inverse problem. We need to design a voltage signal for a given power spectrum. We use the spectrum (39), obtained from earlier calculated components (37) and (38), as such power spectrum. Numerical values (37), (38) can be obtained either by the substitution numerical values of amplitudes (35), (36) or by using the conversion, inverse to (2):

$$P=\begin{pmatrix}P_0\\P_1\\P_2\\P_3\\P_4\\P_5\\P_6\\P_7\\P_8\\P_9\\P_{10}\end{pmatrix}=\begin{pmatrix}780,0\\0\\135,82-j4,397\cdot10^3\\0\\277,967+j3,983\cdot10^3\\0\\815,581+j5,931\cdot10^3\\0\\0\\0\\-283,332-1,098\cdot10^3\end{pmatrix}. \qquad (39)$$

Since $\det I_\Delta=-2,295\cdot10^{13}+j2,253\cdot10^{14}\neq0$, the problem has a solution and it is unique. Then, with the substitution of numerical values of amplitudes get:

$$U=\begin{pmatrix}
I_5^* & 0 & 0 & 0 & I_1^* & 0 & I_{-1} & 0 & 0 & 0 & I_{-5}\\
0 & 0 & 0 & I_1^* & 0 & I_{-1} & 0 & 0 & 0 & I_{-5} & 0\\
0 & 0 & I_1^* & 0 & I_{-1} & 0 & 0 & 0 & I_{-5} & 0 & 0\\
0 & I_1^* & 0 & I_{-1} & 0 & 0 & 0 & I_{-5} & 0 & 0 & 0\\
I_1^* & 0 & I_{-1} & 0 & 0 & 0 & I_{-5} & 0 & 0 & 0 & 0\\
0 & I_{-1} & 0 & 0 & 0 & I_{-5} & 0 & 0 & 0 & 0 & 0\\
I_{-1} & 0 & 0 & 0 & I_{-5} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & I_{-5} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & I_{-5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & I_{-5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
I_{-5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}^{-1}\begin{pmatrix}P_0\\P_1\\P_2\\P_3\\P_4\\P_5\\P_6\\P_7\\P_8\\P_9\\P_{10}\end{pmatrix}=\begin{pmatrix}-6,816-j55,798\\0\\-1,54\cdot10^{-15}-j6,659\cdot10^{-15}\\0\\-2,889+j49,498\\0\\-2,889-j49,498\\0\\7,105\cdot10^{-14}\\0\\-6,816+j55,798\end{pmatrix}. \qquad (40)$$

One can see from (40), the inverse problem has a small, but insignificant error in terms of the electromechanical problem, caused by a division operation. Error is absent only in those cases where the values of the amplitudes, that involved in the deconvolution, evenly divisible by each other, which is almost impossible in practice.

Thus, presented theoretical relations are a mathematical basis to a unified approach to analysis and synthesis of signals of instantaneous power and voltage and to solution of electromechanical problems, involving approach, that based on instantaneous power in nonsinusoidal circuits.

CONCLUSIONS. Considered a common approach to analysis and synthesis of instantaneous power and voltage signals in non-sinusoidal circuits, which along with the quantitative estimation allow to made the qualitative evaluation of their spectra formation mechanism. The last is actual, in particular, to solve electrical problems.

Obtained mathematical relations are presented in conventional terms of signal theory and matrix algebra. This on practice allows to synthesize efficient computational procedures in terms of performance and is important in design of real-time systems, which is actual in control of quality power conversion.

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**АНАЛИЗ СИГНАЛОВ МГНОВЕННОЙ МОЩНОСТИ И ЕЕ СОСТАВЛЯЮЩИХ В НЕСИНУСОИДАЛЬНЫХ ЦЕПЯХ ПРИ РЕШЕНИИ ЭЛЕКТРОТЕХНИЧЕСКИХ ЗАДАЧ**

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Рассмотрен единый подход к анализу и синтезу сигналов мгновенной мощности и напряжения для несинусоидальных цепей, позволяющий, в отличие от ранее известных, наряду с количественной оценкой сделать качественный анализ механизма формирования их спектров. Рассмотрена задача преобразования спектра несинусоидального периодического сигнала при сдвиге системы координат. Подходы актуальны при решении уравнений энергетического баланса в задачах идентификации параметров электрических машин с учетом их нелинейного характера и при синтезе сигналов напряжения для заданного гармонического состава мгновенной мощности в задачах управления качеством преобразования электрической энергии. Полученные математические соотношения представлены в общепринятых терминах теории сигналов и матричной алгебры. На практике это позволит выполнить синтез быстродействующих вычислительных процедур и является важным при проектировании систем управления качеством преобразования энергии в реальном времени.

**Ключевые слова:** мгновенная мощность, гармонический анализ, свертка, обратная свертка.

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