UDC 620.179:621.373.5

MODELING OF ELECTROMAGNETIC PROCESSES IN ELECTROTECHNICAL COMPLEXES FOR REDUCING RESIDUAL STRESSES

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It was developed the axisymmetric mathematical model for calculating the instantaneous current density in the electrode system, which is part of the electrical complex to reduce the residual stresses. It was determined the instantaneous electromagnetic force affects the non-ferromagnetic conductive disc and consequently the pressure force of the electrode on the non-ferromagnetic conductive plate and the current spreading in the electrode contact area with a non-ferromagnetic conductive plate, calculated density distribution of the magnetic forces and the contact pressure zone. The resulting electromagnetic field is the superposition of electromagnetic fields was found.The distribution of current density and electromagnetic forces which is a prerequisite to the explanation of the phenomenon changes of residual stresses in the contact area of the electrode with a non-ferromagnetic conductive plate was analyzed.

Key words: residual stresses, electrode system, impuls current, Maxwell's equations, integral equation method, electrodynamic forces.

МОДЕЛЮВАННЯ ЕЛЕКТРОМАГНІТНИХ ПРОЦЕСІВ В ЕЛЕКТРОТЕХНІЧНОМУ КОМПЛЕКСІ ДЛЯ ЗНИЖЕННЯ ЗАЛИШКОВИХ НАПРУЖЕНЬ

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Розроблено вісесиметричну математичну модель розрахунку миттєвої густини струму в електродній системі, що є складовою частиною електротехнічного комплексу для зниження залишкових напружень. Визначено миттєву електромагнітну силу, що діє на неферомагнітний провідний диск, і, як наслідок, силу тиску електрода на неферомагнітну провідну пластину та струм розтікання в зоні контакту електрода з неферомагнітною провідною пластиною. Розраховано розподіл густини магнітних сил та тиску в зоні контакту. Знайдено результуюче електромагнітне поле накладенням електромагнітних полів. Проаналізовано розподіл густини струму та електромагнітних сил, що є передумовою до пояснення явища зміни залишкових напружень у зоні контакту електрода з неферомагнітною провідною пластиною.

Ключові слова: залишкові напруження, електродна система, імпульс струму, рівняння Максвелла, метод інтегральних рівнянь, електродинамічні сили.

PROBLEM STATEMENT. Residual stresses (RS) is the elastic strain and the corresponding stress which is balanced within the body in the absence of external forces. They always appear at the end of any technological processes, production, strengthening and restoration parts and components. Opportunities level control RS with technological methods are limited, and usually their level is still quite high even at the optimum manufacturing technology.

Thus, the problem of reducing residual stresses during the technological process of production and exploitation is relevant to ensure the reliability required characteristics, durability, operability and safety equipment in various branches of agriculture.

One of the method to reducing RS is an additional action on metal by electric current with the density that is exceeded a certain threshold (hundreds A/mm^2).

Despite the extensive evidence in this area there is no information about the parameters current pulses at which the local relaxation of residual stresses is achieved.

In the article an electrotechnical complex for reducing residual stresses which consists of two independent circuits – circuit 1 and circuit 2, each of which consists of series-connected capacity, inductance and active resistance (Fig. 1) is engineered. The first circuit is intended for providing of given pressure forces on non-ferromagnetic conductive disc D_1 which is rigidly jointed with the electrode D_2. The second one enables setting the current pulse in the sample test.

In general, the problem requires Maxwell's equations solving in a three-dimensional region, but if we do some assumptions, the problem can be reduced to the two-dimensional problem. That is, if the coil, electrode and plate are massive cylindrical body having a common rotation axis, which later is combined with *z*-axis of cylindrical coordinate system r, α, z (Fig. 1), the problem is considered in axisymmetrical formulation [1–4].

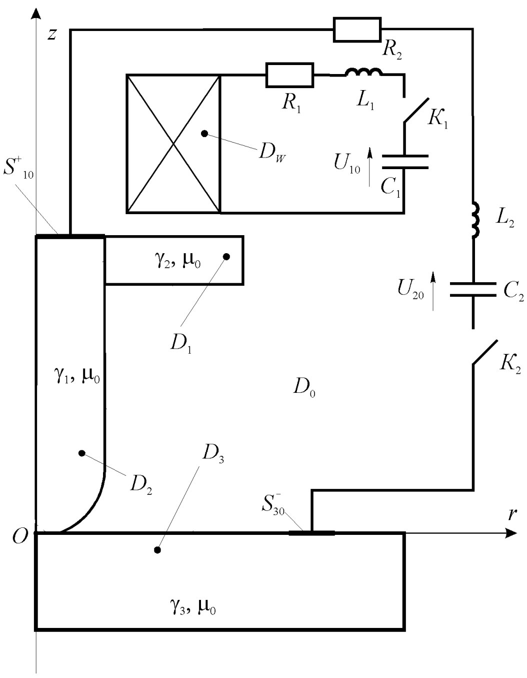

Figure 1 – Electrotechnical complexes for reduce residual stresses

On Fig. 1 R_1, L_1, C_1 are the electrical parameters of first circuit; U_{10} is capacitor C_1 voltage of first circuit; R_2, L_2, C_2 are electrical parameters of second circuit; U_{20} is capacitor C_2 voltage of second circuit; D_1 is nonferromagnetic conductive disc; D_2 is nonferromagnetic electrodes; D_3 is nonferromagnetic conducting plate, in which we should reduce the residual stress; D_0 is external space; D_w is inductance coil; K_1 is the first thyristor circuit; K_2 is second circuit thyristor.

The aim of the article is to calculate the current $i(t)$ in the capacitor C_2 discharge circle, the current density distribution $\delta(Q,t)$ in massive conductors, electromagnetic force $F(t)$ that is affects the electrode with disk and so on for a given system geometry, electrophysical characteristics of the structural elements material, electrical elements connection, given capacitor voltage.

The original problem is formed as a combination of two subtasks.

1. We assume that geometric parameters of the electrode system and the electrical properties of materials, instant current density in the coil D_W that has only angular components is given.

2. We assume that geometric parameters of the electrode system and the electrical properties of materials, instantaneous current density at the contact junction S^+_{10}, S^-_{30} of the external circuit with the electrode system (electrode, nonferromagnetic conductive plate).

The first subtask allows calculating the instantaneous electromagnetic force that affects the nonferromagnetic conductive disc D_1 and therefore the pressure force on the electrode nonferromagnetic conductive plate D_3.

The second subtask allows calculating the current spreading at the contact junction of electrode D_2 with nonferromagnetic conductive plate D_3.

EXPERIMENTAL PART AND RESULTS OBTAINED. The resulting magnetic field is calculated by superposition of electromagnetic fields that are separately calculated in each subtask [5, 6].

In the first subtask, there are the following expressions for the components of the electromagnetic field and the eddy currents in massive conductors:

$$\vec{E} = \vec{e}_\alpha E(r,z,t); \tag{1}$$

$$\vec{\delta}(r,z,t) = \delta_\alpha(r,z,t)\vec{e}_\alpha. \tag{2}$$

$$\vec{A}(r,z,t) = A_\alpha(r,z,t)\vec{e}_\alpha; \tag{3}$$

$$\vec{B}(r,z,t) = B_r(r,z,t)\vec{e}_r + B_z(r,z,t)\vec{e}_z; \tag{4}$$

in the second subtask:

$$\vec{E}(r,z,t) = E_r(r,z,t)\vec{e}_r + E_z(r,z,t)\vec{e}_z; \tag{5}$$

$$\vec{\delta}(r,z,t) = \delta_r(r,z,t)\vec{e}_r + \delta_z(r,z,t)\vec{e}_z. \tag{6}$$

$$\vec{A}(r,z,t) = A_r(r,z,t)\vec{e}_r + A_z(r,z,t)\vec{e}_z; \tag{7}$$

$$\vec{B}(r,z,t) = B_\alpha(r,z,t)\vec{e}_\alpha. \tag{8}$$

For the task solutions we formulate a boundary-value problem in terms of the magnetic vector potential and electric scalar potential that reduces to an integro-differential equations system (space integration variables, time differential) with the use of potential theory.

These systems of integro-differential equations are solved using the approximation of the spatial variables by complete averaging method and the approximation of the time variable by first or second order difference schemes [7].

After the density currents in conductors will be calculated, the force affecting the disk, and the electromagnetic forces distribution in the contact zone of the electrode D_1 and nonferromagnetic conducting plate D_3 are calculated. Analysis of the current density distribution and electromagnetic forces is a precondition for explanation of residual stress effects in the contact zone of the electrode with nonferromagnetic conductive plate.

At the beginning the first subtask is considered. After integration over the azimuthal angle the vector potential expression takes the following form

$$A_\alpha(Q,t) = \frac{\mu_0}{2\pi}\int_{S_W} \delta_{\alpha W}(M,t)T(Q,M)dS_M + \frac{\mu_0}{2\pi}\int_S \delta_\alpha(M,t)T(Q,M)dS_M, \tag{9}$$

where S is the meridian cross-section area of massive conductors; S_W is same for the coil;

$$T(Q,M)=\sqrt{\frac{r_M}{r_Q}}f(k); \qquad (10)$$

$f(k)$ is formed by the expression

$$f(k)=\left(\frac{2}{k}-k\right)K(k)-\frac{2}{k}E(k); \qquad (11)$$

$$k^2=\frac{4r_Q r_M}{(r_Q+r_M)^2+(z_Q-z_M)^2}; \qquad (12)$$

$$K(k)=\int_0^{\pi/2}\frac{d\alpha}{\sqrt{1-k^2\sin^2\alpha}};$$

$$E(k)=\int_0^{\pi/2}\sqrt{1-k^2\sin^2\alpha}\,d\alpha \qquad (13)$$

– is complete elliptic integrals of the first and second kind of module k; r_M, z_M is the position of the point M in a cylindrical coordinate system; r_Q, z_Q is similar for the point Q.

The components of magnetic induction $\vec{B}$ can be found as follows:

$$B_r(Q,t)=\frac{\mu_0}{2\pi}\int_S\frac{\delta(M,t)(z_Q-z_M)}{r_Q\sqrt{(r_Q+r_M)^2+(z_Q-z_M)^2}}\times$$
$$\times\left[-K(k)+\frac{r_Q^2+r_M^2+(z_Q-z_M)^2}{(r_M-r_Q)^2+(z_Q-z_M)^2}E(k)\right]dS_M;$$
$$B_z(Q,t)=\frac{\mu_0}{2\pi}\int_S\frac{\delta(M,t)}{\sqrt{(r_Q+r_M)^2+(z_Q-z_M)^2}}\times \qquad (14)$$
$$\times\left[K(k)+\frac{r_M^2-r_Q^2-(z_Q-z_M)^2}{(r_Q-r_M)^2+(z_Q-z_M)^2}E(k)\right]dS_M.$$

We introduce the following notation

$$b_r(Q,M)=\frac{(z_Q-z_M)}{r_Q\sqrt{(r_Q+r_M)^2+(z_Q-z_M)^2}}\times \qquad (15)$$
$$\times\left[-K(k)+\frac{r_Q^2+r_M^2+(z_Q-z_M)^2}{(r_M-r_Q)^2+(z_Q-z_M)^2}E(k)\right];$$

$$b_z(Q,M)=\frac{1}{\sqrt{(r_Q+r_M)^2+(z_Q-z_M)^2}}\times \qquad (16)$$
$$\times\left[K(k)+\frac{r_M^2-r_Q^2-(z_Q-z_M)^2}{(r_Q-r_M)^2+(z_Q-z_M)^2}E(k)\right];$$

$$\vec{b}(Q,M)=b_r(Q,M)\vec{e}_r+b_z(Q,M)\vec{e}_z. \qquad (17)$$

Then the expression for calculating the magnetic field can be written as

$$\vec{B}(Q,t)=\frac{\mu}{2\pi}\int_D\delta(M,t)\vec{b}(Q,M)dS_M. \qquad (18)$$

Ohm's law is performed in massive conductors sections:

$$\delta_\alpha(Q,t)=\gamma E_\alpha(Q,t),\ Q\in D. \qquad (19)$$

From Maxwell's equations $rot\vec{E}=-\partial\vec{B}/\partial t$ and relation $\vec{B}=rot\vec{A}$ it follows

$$\vec{E}=-\frac{\partial\vec{A}}{\partial t}-grad\varphi,$$

where φ is the electric scalar potential. It can be shown that the first task in the axisymmetrical case $grad\varphi=0$, therefore in the massive conductors sections

$$E_\alpha(Q,t)=-\frac{\partial A_\alpha(Q,t)}{\partial t}. \qquad (20)$$

Substituting in the relation (20) the expression for the vector potential in the form (9), then the resulting expression for the electric field in the relation (19), we obtain the following system of integro-differential equations for calculating the eddy currents density in massive conductors:

$$\frac{\delta_\alpha(Q,t)}{\gamma\lambda_S}+\frac{\partial}{\partial t}\int_S\delta_\alpha(M,t)T(Q,M)dS_M= \qquad (21)$$
$$=-\frac{\partial}{\partial t}\int_{S_W}\delta_{W\alpha}(M,t)T(Q,M)dS_M,\ Q\in D.$$

Thus, for calculating α-component eddy currents in massive conductors caused by currents in the coil D_W enough to solve the system of integral-differential equations (21).

Let us consider the second subtask. It is not difficult to show angular component of magnetic induction in a cylindrical coordinate system. In this case it is

$$B_\alpha(Q,t)=\frac{\mu_0}{2\pi}\int_S\delta_r(M,t)b_r(Q,M)dS_M- \qquad (22)$$
$$-\frac{\mu_0}{2\pi}\int_S\delta_z(M,t)b_z(Q,M)dS_M,$$

where $b_r(Q,M)$, $b_z(Q,M)$ are defined by expressions (15) and (16).

After integration over the azimuthal angle, the scalar electric potential takes the form

$$\varphi(Q,t)=\frac{1}{2\pi\varepsilon_0}\int_l\sigma(M,t)\sqrt{\frac{r_M}{r_Q}}kK(k)dl_M. \qquad (23)$$

The vector of the electric field

$$\vec{E}(Q,t)=-grad\varphi=-\frac{1}{2\pi\varepsilon_0}\times \qquad (24)$$
$$\times\int_L\sigma(M,t)\vec{\xi}(Q,M)dL_M,$$

where

$$\vec{\xi}(Q,M)=\xi_r(Q,M)\vec{e}_r+\xi_z(Q,M)\vec{e}_z; \qquad (25)$$

$$\xi_r(Q,M)=\frac{r_M}{r_Q\sqrt{(r_Q+r_M)^2+(z_M-z_Q)^2}}\times$$
$$\times\left[\frac{r_M^2-r_Q^2+(z_M-z_Q)^2}{(r_Q-r_M)^2+(z_M-z_Q)^2}E(k)-K(k)\right]; \quad (26)$$

$$\xi_z(Q,M)=\frac{2r_M(z_M-z_Q)}{\sqrt{(r_M+r_Q)^2+(z_M-z_Q)^2}}\times$$
$$\times\frac{1}{\left[(r_M-r_Q)^2+(z_M-z_Q)^2\right]}E(k). \quad (27)$$

Thus, the system of integro-differential equations in a cylindrical coordinate system takes the form

$$\frac{\vec\delta(Q,t)}{\gamma\lambda_L}+\frac{\partial}{\partial t}\int_S\vec\delta(M,t)T(Q,M)\,dS_M-\frac{1}{\varepsilon_0\mu_0}\times$$
$$\times\int_L\sigma(M,t)\vec\xi\,dL_M=-\frac{\partial}{\partial t}\int_{S_W}\vec\delta_W(M,t)T(Q,M)\,dS_M, \quad (28)$$
$$Q\in S;$$

$$\frac{\chi(Q)\mu_0\varepsilon_0}{\pi}\frac{\partial}{\partial t}\int_S\left(\vec\delta(M,t),\vec n_Q\right)T(Q,M)dS_M+$$
$$+\sigma(Q,t)+\frac{\chi(Q)}{\pi}\int_L\sigma(M,t)\left(\vec\xi(Q,M),\vec n_Q\right)dL_M= \quad (29)$$
$$=-\frac{\chi(Q)\mu_0\varepsilon_0}{\pi}\frac{\partial}{\partial t}\int_{S_W}\left(\vec\delta_W(M,t),\vec n_Q\right)T(Q,M)\times$$
$$\times dS_M-F(Q,t),Q\in L\cup L_{10}^{+}\cup L_{30}^{-},$$

where $\lambda_L=\mu_0/(2\pi)$; L is the trace of the solid conductor border S cross-section by the meridian area.

Instantaneous three-dimensional electrodynamic forces density $\vec f(Q,t)$ in the cross-section of the conducting plate with current density $\vec\delta(Q,t)$, which is in the magnetic field $\vec B(Q,t)$ caused by the currents in the coil and eddy currents in massive conductors is defined as follows

$$\vec f(Q,t)=\left[\vec\delta(Q,t),\vec B(Q,t)\right].$$

Whereas the current density $\vec\delta(Q,t)$ has been represented in the form (6) and the magnetic induction vector has only α -component (8) we derive the expression for the vector

$$\vec F(Q,t)=-\vec e_r\delta_z(Q,t)B_\alpha(Q,t)+$$
$$+\vec e_z\delta_z(Q,t)B_\alpha(Q,t). \quad (30)$$

Magnetic induction is calculated according to the expression (22).

One of the most widespread and practically high-used numerical methods of the system of the integro-differential equations allows reducing them to the system of linear algebraic equations (SLAE) [8–11].

Let's do on N_{S_q} constant element discretization of areas S_q, $q=1,2,...,N$, which is occupied by the massive conductors of the electrosystem magnetic "electrode-disk-plate", with elementary areas $\Delta S_q^i, i=1,2,...,N_{S_q}$, by orthogonal grids; a coil area S_W is discretizated with N_{S_W} elementary areas ΔS_W^i $i=1,2,...,N_{S_W}$. The contour of massive conductors section L is discretizated with N_L linear elements ΔL^i $i=1,2,...,N_L$.

Applying a full averaging method we shall receive system of the linear algebraic equations which approximates system of the integrated equations (21) on spatial variables:

$$\frac{1}{\gamma_q\lambda_S}\Delta\mathrm{S}_q\vec\delta_{\alpha q}(t)+\frac{\partial}{\partial t}\sum_{\substack{m=1\\q\ne m}}^{3}T_{S_qS_m}\vec\delta_{\alpha m}(t)=$$
$$=-\frac{\partial}{\partial t}T_{S_qS_W}\vec\delta_{W\alpha}(t),$$

where

$$\vec\delta_{\alpha m}(t)=\left\|\delta_{\alpha m1}(t),\delta_{\alpha m2}(t),...,\delta_{\alpha mN_{S_m}}(t)\right\|^T$$

– a column vector elements of which are instant values of an α component of eddy currents density in the center of elementary area ΔS_m^i $i=1,2,...,N_{S_m}$;

$$\vec\delta_{W\alpha}(t)=\left\|\delta_{W\alpha1}(t),\delta_{W\alpha2}(t),...,\delta_{W\alpha N_{S_W}}(t)\right\|^T$$

– a column vector the elements of which are instant values of currents density in the center of elementary area ΔS_w^i, $i=1,2,...,N_{S_W}$ into which the area of the coil is discretizated;

$\mathbf{\Delta S}_q=diag\left\|\Delta S_q^1,\Delta S_q^2,...,\Delta S_q^{N_{S_q}}\right\|$ – a diagonal matrix the elements of which are the areas ΔS_q^i, $q=1,2,3$, $i=1,2,...,N_{S_q}$, of elementary areas into which the q – massive conductor section is discretizated;

$$\mathbf{T}_{S_qS_m}=\left\|\int_{\Delta S_q^i}\int_{\Delta S_m^j}T(Q,M)\,dS_M dS_Q\right\|, i=1,2,...,N_{S_q},$$
$j=1,2,...,N_{S_m}$, $q,m=1,2,3$;

$$\mathbf{T}_{S_qS_W}=\left\|\int_{\Delta S_q^i}\int_{\Delta S_W^j}T(Q,M)\,dS_M dS_Q\right\|, i=1,2,...,N_{S_q},$$
$$q=1,2,3,\ j=1,2,...,N_{S_W}.$$

The system of the equations (28)–(29) on spatial variables is approximated by the following system of the equations:

$$\frac{\mathbf{\Delta S}_q\vec\delta_{rq}(t)}{\gamma_q\lambda_L}+\frac{\partial}{\partial t}\sum_{m=1}^{3}\mathbf{T}_{S_qS_m}\vec\delta_{rm}(t)-\frac{1}{\varepsilon_0\mu_0}\mathbf{\Theta}_{S_qL}^r\vec\sigma(t)=0,$$
$$q=1,2,3; \quad (31)$$

$$\frac{\mathbf{\Delta S}_q\vec{\delta}_{zq}(t)}{\gamma_q\lambda_L}+\frac{\partial}{\partial t}\sum_{m=1}^{3}\mathbf{T}_{S_qS_m}\vec{\delta}_{zm}(t)-\frac{1}{\varepsilon_0\mu_0}\mathbf{\Theta}^z_{S_qL}\vec{\sigma}(t)=0,$$
$$q=1,2,3, \qquad (32)$$

$$\frac{\mu_0\varepsilon_0}{\pi}\frac{\partial}{\partial t}\sum_{m=1}^{3}\mathbf{\chi}_{LL}\mathbf{n}^r_{LL}\mathbf{T}_{LS_m}\vec{\delta}_{rm}(t)+$$
$$+\frac{\mu_0\varepsilon_0}{\pi}\frac{\partial}{\partial t}\sum_{m=1}^{3}\mathbf{\chi}_{LL}\mathbf{n}^z_{LL}\mathbf{T}_{LS_m}\vec{\delta}_{zm}(t)+ \qquad (33)$$
$$+\mathbf{\Delta L}\vec{\sigma}(t)+\frac{1}{\pi}\mathbf{\chi}_{LL}\mathbf{\Theta}_{LL}\vec{\sigma}(t)=\mathbf{\Delta L}\vec{F}_L(t),$$

where

$$\vec{\delta}_{rm}(t)=\left\|\delta_{rm1}(t),\delta_{rm2}(t),...,\delta_{rmN_{S_m}}(t)\right\|^T;$$

$\vec{\delta}_{zm}(t)=\left\|\delta_{zm1}(t),\delta_{zm2}(t),...,\delta_{zmN_{S_m}}(t)\right\|^T$ – a column vectors the elements of which are instant values r - and z – component of eddy current densities in the center of elementary area ΔS^i_m $i=1,2,...,N_{S_m}$; $\vec{\sigma}(t)=\left\|\sigma_1(t),\sigma_2(t),...,\sigma_{N_L}\right\|$ – a column vectors the elements of which are instant values of surface electric charge densities on elementary line ΔL^i $i=1,2,...,N_L$; $\mathbf{\chi}_{LL}=diag\left\|\chi_1,\chi_2,...\chi_{N_L}\right\|$ – a diagonal matrix of dimension $N_L\times N_L$ the elements of which are values of function $\chi(Q)$ in the central points Q_i, $i=1,2,...,N_L$, of elementary areas on which a interface L between mass conductors and also between massive conductors and external space is discretizated; $\mathbf{n}^r_{LL}=diag\left\|n_{r1},n_{r2},...n_{rN_L}\right\|$, $\mathbf{n}^z_{LL}=diag\left\|n_{z1},n_{z2},...n_{zN_L}\right\|$ – a diagonal matrixes by dimension $N_L\times N_L$ the elements of which is r - and z -components of a normal to elementary border ΔL^i of a massive conductor; $\mathbf{\Delta L}=diag\left\|\Delta L^1,\Delta L^2,...,\Delta L^{N_L}\right\|$ – a diagonal matrix the elements of which are lengths of elementary line ΔL^i, $i=1,2,...,N_L$, into which the border L is discretizated; $\vec{F}_L(t)=\left\|\vec{F}_L(Q_1,t),\vec{F}_L(Q_2,t),...,\vec{F}_L(Q_{N_L},t)\right\|^T$ – a column vector the elements of which are values of function $F(Q,t)$ in the central points Q_i, $i=1,2,...,N_L$, of linear elements into which the border L is discretizated.

In the equations (31)–(33) are accepted the following designations for matrixes

$$\mathbf{T}_{S_qS_m}=\left\|\int_{\Delta S^i_q}\int_{\Delta S^j_m}T(Q,M)\,dS_M\,dS_Q\right\|,i=1,2,...,N_{S_q},$$
$$j=1,2,...,N_{S_m},\ q,m=1,2,3;$$

$$\mathbf{\Theta}^r_{S_qL}=\left\|\int_{\Delta S^i_q}\int_{\Delta L^j}\xi_r(Q,M)\,dL_M\,dS_Q\right\|,i=1,2,...,N_{S_q},$$
$$q=1,2,3,\ j=1,2,...,N_L;$$

$$\mathbf{\Theta}^z_{S_qL}=\left\|\int_{\Delta S^i_q}\int_{\Delta L^j}\xi_z(Q,M)\,dL_M\,dS_Q\right\|,i=1,2,...,N_{S_q},$$
$$q=1,2,3,\ j=1,2,...,N_L;$$

$$\mathbf{T}_{LS_m}=\left\|\int_{\Delta L^i}\int_{\Delta S^j_m}T(Q,M)\,dS_M\,dL_Q\right\|,j=1,2,...,N_{S_m},$$
$$m=1,2,3,\ i=1,2,...,N_L;$$

$$\mathbf{\Theta}_{LL}=\left\|\int_{\Delta L^i}\int_{\Delta L^j}\left(\vec{\xi}(Q,M),\vec{n}_Q\right)dL_M\,dL_Q\right\|,i=1,2,...,N_L,$$
$$j=1,2,...,N_L.$$

The first variants of difference schemes for the solution of the similar equations have been described in work [12]. In works [13–15] total schemes distinctive feature of which is that the original differential equations are not approximated but equivalent to them Volterra integral equations are approximated. These equations are formed after integration the original differential equations in an interval $[t_0, t]$.

On time axis we choose generally a non-uniform grid $t_n=n\Delta t_n$, $n=1,2,3,...$, where Δt_n – a step of a time grid. The index n is the index of time a layer. Let the problem is solved on a layer t_{n-1}. To receive difference scheme we shell integrate the equation (31)–(33) on time in an interval $\left[t_{n-1}, t_n\right]$. Then for calculation of integrals we use a relation

$$\int_{t_{n-1}}^{t_n}f(t)\,dt=\left[cf^{(n)}+(1-c)f^{(n-1)}\right]\Delta t_n, \qquad (34)$$

where value of function $f^{(n)}$, $f^{(n-1)}$ is corresponding to the time moments t_n t_{n-1}; c – a weight factor which accepts value from set $\{0;\ 0{,}5;\ 1\}$. At $c=0$ and $c=1$ the formula is the formula of rectangulars, and at $c=0{,}5$ the formula is the formula of trapezes.

Having executed simple transformations, we write down system of the algebraic equations which approximates the differential equations system (31)–(33) as

$$\left(\frac{\mathbf{\Delta S}_q}{\gamma_q\lambda_L}+c\Delta t_n\mathbf{T}_{S_qS_q}\right)\vec{\delta}^{(n)}_{rq}+c\Delta t_n\sum_{\substack{m=1\\m\neq q}}^{3}\mathbf{T}_{S_qS_m}\vec{\delta}^{(n)}_{rm}-$$
$$-\frac{1}{\varepsilon_0\mu_0}\mathbf{\Theta}^r_{S_qL}\vec{\sigma}^{(n)}=-(1-c)\Delta t_n\sum_{m=1}^{3}\mathbf{T}_{S_qS_m}\vec{\delta}^{(n-1)}_{rm},$$
$$q=1,2,3; \qquad (35)$$

$$\left(\frac{\mathbf{\Delta S}_q}{\gamma_q\lambda_L}+c\Delta t_n\mathbf{T}_{S_qS_q}\right)\vec{\delta}^{(n)}_{zq}+c\Delta t_n\sum_{\substack{m=1\\m\neq q}}^{3}\mathbf{T}_{S_qS_m}\vec{\delta}^{(n)}_{zm}-$$
$$-\frac{1}{\varepsilon_0\mu_0}\mathbf{\Theta}^z_{S_qL}\vec{\sigma}^{(n)}=-(1-c)\Delta t_n\sum_{m=1}^{3}\mathbf{T}_{S_qS_m}\vec{\delta}^{(n-1)}_{zm},$$
$$q=1,2,3; \qquad (36)$$

$$\frac{\mu_0\varepsilon_0}{\pi}c\Delta t_n\sum_{m=1}^{3}\boldsymbol{\chi}_{LL}\mathbf{n}_{LL}^{r}\mathbf{T}_{LS_m}\vec{\delta}_{rm}^{(n)}+$$
$$+\frac{\mu_0\varepsilon_0}{\pi}c\Delta t_n\sum_{m=1}^{3}\boldsymbol{\chi}_{LL}\mathbf{n}_{LL}^{z}\mathbf{T}_{LS_m}\vec{\delta}_{zm}^{(n)}+$$
$$+\left(\mathbf{\Delta L}+\frac{1}{\pi}\mathbf{\Theta}_{LL}\right)\vec{\sigma}^{(n)}=$$
$$=\mathbf{\Delta L}\vec{F}_L^{(n)}-\frac{\mu_0\varepsilon_0}{\pi}(1-c)\Delta t_n\sum_{m=1}^{3}\boldsymbol{\chi}_{LL}\mathbf{n}_{LL}^{r}\mathbf{T}_{LS_m}\vec{\delta}_{rm}^{(n-1)}-$$
$$-\frac{\mu_0\varepsilon_0}{\pi}(1-c)\Delta t_n\sum_{m=1}^{3}\boldsymbol{\chi}_{LL}\mathbf{n}_{LL}^{z}\mathbf{T}_{LS_m}\vec{\delta}_{zm}^{(n-1)}. \quad (37)$$

Let's express from the equation (38) $\vec{\sigma}^{(n)}$ and we shall substitute the derived expression at the equation (35), (36):

$$\vec{\sigma}^{(n)}=\tilde{\Theta}_{LL}\mathbf{\Delta L}\vec{F}_L^{(n)}-\frac{\mu_0\varepsilon_0}{\pi}(1-c)\Delta t_n\times$$
$$\times\sum_{m=1}^{3}\tilde{\mathbf{\Theta}}_{LL}\boldsymbol{\chi}_{LL}\mathbf{n}_{LL}^{r}\mathbf{T}_{LS_m}\vec{\delta}_{rm}^{(n-1)}-$$
$$-\frac{\mu_0\varepsilon_0}{\pi}(1-c)\Delta t_n\sum_{m=1}^{3}\tilde{\mathbf{\Theta}}_{LL}\boldsymbol{\chi}_{LL}\mathbf{n}_{LL}^{z}\mathbf{T}_{LS_m}\vec{\delta}_{zm}^{(n-1)}-$$
$$-\frac{\mu_0\varepsilon_0}{\pi}c\Delta t_n\sum_{m=1}^{3}\tilde{\mathbf{\Theta}}_{LL}\boldsymbol{\chi}_{LL}\mathbf{n}_{LL}^{r}\mathbf{T}_{LS_m}\vec{\delta}_{rm}^{(n)}-$$
$$-\frac{\mu_0\varepsilon_0}{\pi}c\Delta t_n\sum_{m=1}^{3}\tilde{\mathbf{\Theta}}_{LL}\boldsymbol{\chi}_{LL}\mathbf{n}_{LL}^{z}\mathbf{T}_{LS_m}\vec{\delta}_{zm}^{(n)}, \quad (38)$$

where

$$\tilde{\Theta}_{LL}=\left(\mathbf{\Delta L}+\frac{1}{\pi}\mathbf{\Theta}_{LL}\right)^{-1}.$$

Next, substituting the equation (38) in the equations (35), (36) we receive

$$\left(\frac{\mathbf{\Delta S}_q}{\gamma_q\lambda_L}+c\Delta t_n\tilde{\mathbf{T}}_{S_qS_q}^{rr}\right)\vec{\delta}_{rq}^{(n)}+c\Delta t_n\sum_{\substack{m=1\\m\neq q}}^{3}\tilde{\mathbf{T}}_{S_qS_m}^{rr}\vec{\delta}_{rm}^{(n)}+$$
$$+c\Delta t_n\sum_{m=1}^{3}\tilde{\mathbf{T}}_{S_qS_m}^{rz}\vec{\delta}_{zm}^{(n)}=$$
$$=\frac{1}{\varepsilon_0\mu_0}\mathbf{\Theta}_{S_qL}^{r}\tilde{\mathbf{\Theta}}_{LL}\mathbf{\Delta L}\vec{F}_L^{(n)}-(1-c)\Delta t_n\sum_{m=1}^{3}\tilde{\mathbf{T}}_{S_qS_m}^{rr}\vec{\delta}_{rm}^{(n-1)}-$$
$$-(1-c)\Delta t_n\sum_{m=1}^{3}\tilde{\mathbf{T}}_{S_qS_m}^{rz}\vec{\delta}_{zm}^{(n-1)}, \quad (39)$$

$$c\Delta t_n\sum_{m=1}^{3}\tilde{\mathbf{T}}_{S_qS_m}^{zr}\vec{\delta}_{rm}^{(n)}+\left(\frac{\mathbf{\Delta S}_q}{\gamma_q\lambda_L}+c\Delta t_n\tilde{\mathbf{T}}_{S_qS_q}^{zz}\right)\vec{\delta}_{zq}^{(n)}+$$
$$+c\Delta t_n\sum_{\substack{m=1\\m\neq q}}^{3}\tilde{\mathbf{T}}_{S_qS_m}^{zz}\vec{\delta}_{zm}^{(n)}=$$
$$=\frac{1}{\varepsilon_0\mu_0}\mathbf{\Theta}_{S_qL}^{r}\tilde{\mathbf{\Theta}}_{LL}\mathbf{\Delta L}\vec{F}_L^{(n)}-(1-c)\Delta t_n\sum_{m=1}^{3}\tilde{\mathbf{T}}_{S_qS_m}^{zr}\vec{\delta}_{rm}^{(n-1)}-$$
$$-(1-c)\Delta t_n\sum_{m=1}^{3}\tilde{\mathbf{T}}_{S_qS_m}^{zz}\vec{\delta}_{zm}^{(n-1)}, \quad (40)$$

where

$$\tilde{\mathbf{T}}_{S_qS_m}^{rr}=\mathbf{T}_{S_qS_m}+\frac{1}{\pi}\mathbf{\Theta}_{S_qL}^{r}\tilde{\mathbf{\Theta}}_{LL}\boldsymbol{\chi}_{LL}\mathbf{n}_{LL}^{r}\mathbf{T}_{LS_m};$$
$$\tilde{\mathbf{T}}_{S_qS_m}^{rz}=\frac{1}{\pi}\mathbf{\Theta}_{S_qL}^{r}\tilde{\mathbf{\Theta}}_{LL}\boldsymbol{\chi}_{LL}\mathbf{n}_{LL}^{z}\mathbf{T}_{LS_m};$$
$$\tilde{\mathbf{T}}_{S_qS_m}^{zr}=\frac{1}{\mu_0}\mathbf{\Theta}_{S_qL}^{r}\tilde{\mathbf{\Theta}}_{LL}\boldsymbol{\chi}_{LL}\mathbf{n}_{LL}^{z}\mathbf{T}_{LS_m};$$
$$\mathbf{T}_{S_qS_m}^{zz}=\mathbf{T}_{S_qS_m}+\frac{1}{\pi}\mathbf{\Theta}_{S_qL}^{z}\tilde{\mathbf{\Theta}}_{LL}\boldsymbol{\chi}_{LL}\mathbf{n}_{LL}^{z}\mathbf{T}_{LS_m}.$$

The system of the algebraic equations (39), (40) allows to pass from a time layer $(n-1)$ to n.

CONCLUSION. The axisymmetrical mathematical model for calculating the instantaneous current density in the electrode system that is part of the electrical complex for reduce residual stresses is developed.

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МОДЕЛИРОВАНИЕ ЭЛЕКТРОМАГНИТНЫХ ПРОЦЕССОВ В ЭЛЕКТРОТЕХНИЧЕСКОМ КОМПЛЕКСЕ ДЛЯ СНИЖЕНИЯ ОСТАТОЧНЫХ НАПРЯЖЕНИЙ

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Разработана осесимметричная математическая модель расчета мгновенной плотности тока в электродной системе, которая является составной частью электротехнического комплекса для снижения остаточных напряжений. Определены мгновенная электромагнитная сила, действующая на неферромагнитный проводящий диск и, как следствие, силу давления электрода на неферромагнитную проводящую пластину и ток растекания в зоне контакта электрода с неферромагнитной проводящей пластиной, рассчитано распределение плотности магнитных сил и давления в зоне контакта. Найдено результирующее электромагнитное поле наложением электромагнитных полей. Проанализировано распределение плотности тока и электромагнитных сил, что является предпосылкой к объяснению явления изменения остаточных напряжений в зоне контакта электрода с неферромагнитной проводящей пластиной.

Ключевые слова: остаточные напряжения, электродная система, импульс тока, уравнения Максвелла, метод интегральных уравнений, электродинамические силы.

Стаття надійшла 23.07.2014.