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DECOMPOSITION AND LINEARIZATION OF ASYNCHRONOUS MOTOR MATHEMATICAL MODELS IN RELATIVE UNITS

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Purpose. To obtain the mathematical expressions for the coefficients of linearized asynchronous motor equations in relative units, which allow making correct compensation of the separate components of the control object dynamic properties by means of inverse dynamic models. **Methodology.** The proposed mathematical transformations and numerical calculations are performed using mathematical models of an asynchronous motor in two-phase orthogonal coordinate systems oriented along the vectors of the rotor-stator flux linkage. To enable the synthesis of control systems, which are built on the principles of solving inverse dynamics problems, a system of relative units with equal mutual inductances between stator and rotor windings, as well as between the phase windings of the rotor is used. **Results.** The asynchronous motor is conditionally divided into two control objects: for the reactive power loop and the active power loop. The determination of control objects models was carried out by using the decomposition and linearization methods of differential equations systems. The obtained coefficients of the linearized equations for asynchronous motor in relative units allow correctly compensating the dynamic properties of separate constituents for control objects of automated electromechanical system by means of inverse dynamic models. **Originality.** The given differential equations and structural block-diagram of the asynchronous motor in two-phase orthogonal coordinate systems oriented by flux linkage vectors provide an exhaustive mathematical description necessary for the implementation of the synthesis of automated electromechanical systems based on a discrete time equalizer. **Practical value.** Expressions for the coefficients of the asynchronous motor equations in relative units, obtained as a result of the transformations performed and entered into a generalization table, allow us to calculate the parameters of the structural block-diagram and mathematical models for a wide range of industrial asynchronous motors. References 10, tables 2, figures 3.

Key words: decomposition, linearization, asynchronous motor.

ДЕКОМПОЗИЦІЯ ТА ЛІНЕАРИЗАЦІЯ МАТЕМАТИЧНИХ МОДЕЛЕЙ АСИНХРОННОГО ДВИГУНА У ВІДНОСНИХ ОДИНИЦЯХ

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Для виконання синтезу сучасних автоматизованих електромеханічних систем з асинхронними двигунами застосовують двофазні математичні моделі, які адекватно відображають процеси, що протікають у реальній машині. Запропоновані в роботі математичні перетворення та чисельні розрахунки виконано із застосуванням математичних моделей асинхронного двигуна у двофазних ортогональних системах координат, орієнтованих за векторами потокозчеплення ротора та статора. Для забезпечення можливості синтезу систем керування, що будуються за принципами розв’язання зворотних задач динаміки, використано систему відносних одиниць з рівними взаємними індуктивностями між статорними й роторними обмотками, а також між фазними обмотками ротора. Асинхронний двигун умовно розділено на два об’єкти керування: для контуру реактивної та контуру активної потужностей. Визначення моделей об’єктів керування здійснено із застосуванням методів декомпозиції та лінеаризації систем диференціальних рівнянь. Отримані коефіцієнти лінеаризованих рівнянь асинхронного двигуна у відносних одиницях дозволяють коректно компенсувати динамічні властивості окремих складових об’єкта керування автоматизованої електромеханічної системи за допомогою обернених динамічних моделей.

Ключові слова: декомпозиція, лінеаризація, асинхронний двигун.

PROBLEM STATEMENT. The total amount of electricity used to set in motion all drives with asynchronous motors accounts for more than 40 % of the total electric power consumption.

Alternating current (AC) drives and electric drives with an asynchronous motor as an executive body in particular have a steady annual growth rate of sales on the world market, while world demand for direct current (DC) drives does not tend to increase. The wide spread occurrence of asynchronous motors in the up-to-date electromechanical systems is due to the simplicity of their design, reliability in operation, low cost and ease of maintenance.

Power converters based on microprocessor technology, which are used in modern AC and DC drives, have reached a very high technical level, which in most cases allows the use of alternating current electric drives in those spheres where previously only regulated direct current drives were used.

The hardware of frequency converters and microcontrollers used in modern automated AC drives is sufficient for the program implementation of regulators constructed by using the principle of symmetry [1] and the concept of inverse dynamics problems for technical objects [2, 3]; in particular regulators constructed using the theory of a discrete time equalizer [4].

Against the background of the current wide spread use of electromechanical systems with asynchronous motors as the executive body, the latter become the most probable control objects for regulators synthesized on the basis of a discrete time equalizer [5].

The relevance of the mathematical models decomposition follows from the necessity of the regulators synthesis for the implementation of vector control of asynchronous motor separately for subsystems of reactive and active powers. In this case, it is necessary to carry out the linearization of mathematical models, since the presence of cross feedbacks leads to the fact that the parameters of the channel of regulation of the magnetic flux linkage also belong to the differential equations that describe the speed control and vice versa.

For correct compensation of the dynamic properties of separate components of the control object with the help of inverse models and mathematical description of perturbed system motion, all calculations are carried out in the system of relative units with equal mutual inductances between the stator and rotor windings, as well as between the phase windings of the rotor [6].

The purpose of the article is obtaining of mathematical expressions for the coefficients of linearized asynchronous motor equations in relative units, which allow making correct compensation of the separate components of the control object dynamic properties by means of inverse dynamic models.

EXPERIMENTAL PART AND RESULTS OBTAINED. Equations (1) [7] allow to execute a mathematical description of an asynchronous motor in two-phase orthogonal coordinate system u, v oriented according to the vector of the rotor flux linkage.

$$\left.\begin{aligned} pI_{su} &= \frac{L_m R_r}{L_r^2 L_s'}\Psi_o - \frac{R_s L_r^2 + R_r L_m^2}{L_r^2 L_s'} I_{su} + \rightarrow \\ &\rightarrow + \left(1/L_s'\right) U_{su} + I_{sv}\omega_{kr}; \\ p\Psi_o &= -\frac{R_r}{L_r}\Psi_o + \frac{R_r L_m}{L_r} I_{su}; \\ pI_{sv} &= -\frac{L_m}{L_r L_s'}\Psi_o\omega - \frac{R_s L_r^2 + R_r L_m^2}{L_r^2 L_s'} I_{sv} + \rightarrow \\ &\rightarrow + \left(1/L_s'\right) U_{sv} - I_{su}\omega_{kr}; \\ p\omega &= \frac{3 z_p L_m}{2 J L_r} I_{sv}\Psi_o - \frac{M_{st}}{J}, \end{aligned}\right\} \quad (1)$$

where p is differentiation operator; ω is angular speed of rotor rotation; $\mathbf{U_s}$ is stator voltage vector with projections U_{su} and U_{sv}; $\mathbf{I_s}$ is stator current vector with projections I_{su} and I_{sv}; R_s, R_r are active stator and rotor windings resistances respectively; z_p is the number of electric machine poles pairs; L_m is mutual inductance of the rotor and stator windings; L_s, L_r are inductances of stator and rotor windings respectively; $L_s' = \left(L_s L_r - L_m^2\right)/L_r$ is transient stator inductance; J is equivalent moment of inertia of electric drive; M_{st} is static load torque reduced to the motor shaft; $\omega_{kr} = \frac{R_r L_m}{L_r \Psi_o} I_{sv} + \omega$ is rotation speed of the coordinate system; Ψ_o is projection of the base flux linkage vector.

In the mathematical description of the asynchronous motor, when the coordinate system is oriented along the base flux linkage vector $\mathbf{\Psi_s}$, the following system of equations is used [6]:

$$\left.\begin{aligned} p\Psi_o &= -I_{su}R_s + U_{su}; \\ pI_{su} &= \frac{R_r}{L_r L_s'}\Psi_o - \frac{R_r L_s + R_s L_r}{L_r L_s'} I_{su} + \rightarrow \\ &\rightarrow + \left(1/L_s'\right) U_{su} - I_{sv}\left(\omega - \omega_{ks}\right); \\ pI_{sv} &= -\frac{R_r L_s + R_s L_r}{L_r L_s'} I_{sv} + \frac{1}{L_s'} U_{sv} - \rightarrow \\ &\rightarrow - \left(1/L_s'\right)\Psi_o\omega + I_{su}\left(\omega - \omega_{ks}\right); \\ p\omega &= \frac{3 z_p}{2J} I_{sv}\Psi_o - \frac{M_{st}}{J}, \end{aligned}\right\} \quad (2)$$

where $\omega_{ks} = \frac{U_{sv} - I_{sv}R_s}{\Psi_o}$ is rotation speed of the coordinate system.

Let us introduce the following basic values:

– current I_{bas}

$$I_{bas} = \sqrt{2} I_{s\,nom}, \quad (3)$$

where $I_{s\,nom}$ is nominal phase current of the stator winding;

– voltage U_{bas}

$$U_{bas} = \sqrt{2} U_{s\,nom}, \quad (4)$$

where $U_{s\,nom}$ is nominal phase voltage of the stator winding;

– frequency ω_{bas}

$$\omega_{bas} = 2\pi f_{psf}, \quad (5)$$

where $f_{psf} = 50\,Hz$ is power supply frequency;

– rotor speed $\omega_{r\,bas}$

$$\omega_{r\,bas} = \omega_{bas}/z_p; \quad (6)$$

– flux linkage Ψ_{bas}

$$\Psi_{bas} = U_{bas}/\omega_{bas}; \quad (7)$$

– torque M_{bas}

$$M_{bas} = 3\Psi_{bas} I_{bas}/2. \quad (8)$$

Taking into account the basic values (3)–(8), let us introduce the relative variables necessary for representing the mathematical models of the asynchronous motor in relative units:

– for the values that characterize the control parameters

$$x_1 = \frac{\Psi_o}{\Psi_{bas}},\ x_2 = \frac{I_{su}}{I_{bas}},\ u_x = \frac{U_{su}}{U_{bas}},\ y_1 = \frac{\omega}{\omega_{r\,bas}},$$

$$y_2 = I_{sv}/I_{bas}\,,\; u_y = U_{sv}/U_{bas}\,; \qquad (9)$$

– for the values that can be considered as disturbing influences

$$f_1 = \begin{cases} \omega_{kr}/\omega_{bas} & when\;\; \mathbf{\Psi}_o = \mathbf{\Psi}_\mathbf{r}; \\ \omega_{ks}/\omega_{bas} & when\;\; \mathbf{\Psi}_o = \mathbf{\Psi}_s. \end{cases} ; \; f_2 = \frac{M_{st}}{M_{bas}}. \qquad (10)$$

Using the relative units (9) and (10) let us convert the equations (1) describing the asynchronous motor in two-phase coordinate system oriented by vector Ψ_r.

Substituting expressions for Ψ_o, I_{su}, U_{su}, ω, I_{sv} U_{sv}, f_1, f_2 from (9) and (10) to (1) and highlighting in their left portions constituents px_1, px_2, py_1 and py_2 let us get following:

$$\left.\begin{aligned} px_2 &= \frac{L_m R_r}{L_r^2 L_s'}\frac{\Psi_{bas}}{I_{bas}}x_1 - \frac{R_s L_r^2 + R_r L_m^2}{L_r^2 L_s'}x_2 + \\ &\quad + \frac{1}{L_s'}\frac{U_{bas}}{I_{bas}}u_x + \omega_{bas} f_1 y_2; \\ px_1 &= -\frac{R_r}{L_r}x_1 + \frac{R_r L_m}{L_r}\frac{I_{bas}}{\Psi_{bas}}x_2; \\ py_2 &= -\frac{L_m}{L_r L_s'}\frac{\omega_{r\,bas}\Psi_{bas}}{I_{bas}}x_1 y_1 - \to \\ &\to -\frac{R_s L_r^2 + R_r L_m^2}{L_r^2 L_s'}y_2 + \frac{1}{L_s'}\frac{U_{bas}}{I_{bas}}u_y - \omega_{bas} f_1 x_2; \\ py_1 &= \frac{3z_p L_m}{2JL_r}\frac{I_{bas}\Psi_{bas}}{\omega_{r\,bas}}x_1 y_2 - \frac{M_{bas}}{J\omega_{r\,bas}}f_2. \end{aligned}\right\} \qquad (11)$$

Let us introduce the following generalizing coefficients:

$$a_{11} = -\frac{R_r}{L_r};\; a_{12} = \frac{R_r L_m}{L_r}\frac{I_{bas}}{\Psi_{bas}};\; a_{21} = \frac{L_m R_r}{L_r^2 L_s'}\frac{\Psi_{bas}}{I_{bas}};$$

$$a_{22} = -\frac{R_s L_r^2 + R_r L_m^2}{L_r^2 L_s'};\; c_2 = \frac{1}{L_s'}\cdot\frac{U_{bas}}{I_{bas}};$$

$$b_{12} = \frac{3z_p L_m}{2JL_r}\frac{I_{bas}\Psi_{bas}}{\omega_{r\,bas}};\; b_{21} = -\frac{L_m}{L_r L_s'}\frac{\omega_{r\,bas}\Psi_{bas}}{I_{bas}};$$

$$b_{22} = -\frac{R_s L_r^2 + R_r L_m^2}{L_r^2 L_s'};\; \zeta_1 = \omega_{bas};\; \zeta_2 = -\frac{M_{bas}}{J\omega_{r\,bas}};$$

$$\zeta_3 = -\omega_{bas}.$$

Then equations (11) can be represented taking into account the generalizing coefficients in the following way:

$$\left.\begin{aligned} px_1 &= a_{11}x_1 + a_{12}x_2; \\ px_2 &= a_{21}x_1 + a_{22}x_2 + c_2 u_x + \zeta_1 f_1 y_2; \\ py_1 &= b_{12}x_1 y_2 + \zeta_2 f_2; \\ py_2 &= b_{21}x_1 y_1 + b_{22}y_2 + c_2 u_y + \zeta_3 f_1 x_2. \end{aligned}\right\} \qquad (12)$$

The rotation speed of the coordinate system when entering relative units

$$\omega_{kr} = \frac{R_r L_m I_{bas}}{L_r \Psi_{bas}}\frac{y_2}{x_1} + \omega_{r\,bas}y_1.$$

Then disturbing actions f_1 and f_2 can be represented by following expressions:

$$\left.\begin{aligned} f_1 &= \frac{\omega_{kr}}{\omega_{bas}} = \zeta_4\frac{y_2}{x_1} + \zeta_5 y_1; \\ f_2 &= M_{st}/\grave{I}_{\;bas}\,, \end{aligned}\right\} \qquad (13)$$

where $\zeta_4 = \dfrac{R_r L_m I_{bas}}{L_r \Psi_{bas}\omega_{bas}}$; $\zeta_5 = \dfrac{\omega_{r\,bas}}{\omega_{bas}} = \dfrac{1}{z_p}$.

Thus, equations (12) and (13) allow executing an exhaustive mathematical description of the asynchronous motor in relative units in two-phase orthogonal coordinate system oriented according to the vector of the rotor flux linkage vector. Values representations are not in absolute terms but in relative units significantly simplify theoretical calculations and gives visibility to the results of calculations. The structural block-diagram of asynchronous motor based on equations (12) and (13) is shown in following figure:

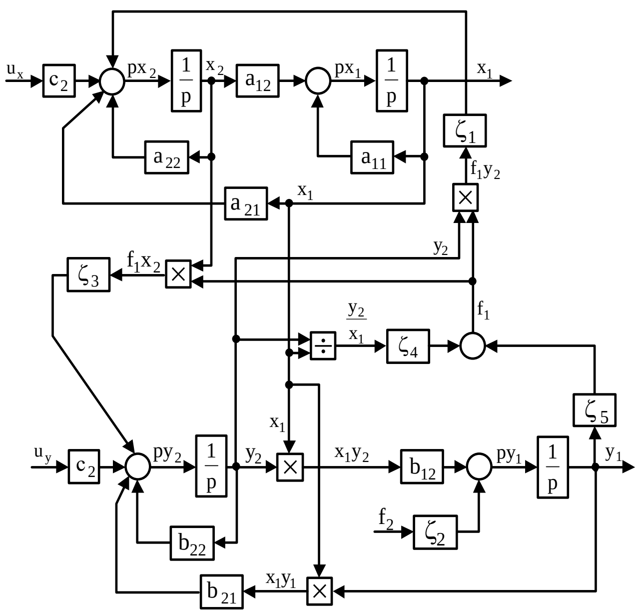

Figure 1 – The structural block-diagram of asynchronous motor in relative units when orienting the coordinate system by vector $\mathbf{\Psi}_\mathbf{r}$

Let us consider the mathematical description of asynchronous motor in relative units in two-phase coordinate system oriented by vector $\mathbf{\Psi}_\mathbf{s}$. To do this in equations (2) the expressions for the parameters Ψ_o, I_{su}, U_{su}, ω, I_{sv} U_{sv}, f_1, f_2 should be substituted by parameters obtained from equations (9) and (10). Highlighting left parts px_1, px_2, py_1 and py_2 gives following expressions:

$$\left.\begin{aligned}
&px_1 = -R_s\frac{I_{bas}}{\Psi_{bas}}x_2 + \frac{U_{bas}}{\Psi_{bas}}u_x;\\
&px_2 = \frac{R_r}{L_rL_s'}\frac{\Psi_{bas}}{I_{bas}}x_1 - \frac{R_rL_s+R_sL_r}{L_rL_s'}x_2 + \rightarrow\\
&\rightarrow + \frac{1}{L_s'}\frac{U_{bas}}{I_{bas}}u_x - \omega_{r\,bas}y_1y_2 + \omega_{bas}f_1y_2;\\
&py_2 = -\frac{R_rL_s+R_sL_r}{L_rL_s'}y_2 + \frac{1}{L_s'}\frac{U_{bas}}{I_{bas}}u_y - \rightarrow\\
&\rightarrow -\frac{1}{L_s'}\frac{\Psi_{bas}\omega_{r\,bas}}{I_{bas}}x_1y_1 + \omega_{r\,bas}x_2y_1 - \omega_{bas}f_1x_2;\\
&py_1 = \frac{3z_p}{2J}\frac{\Psi_{bas}I_{bas}}{\omega_{r\,bas}}x_1y_2 - \frac{M_{bas}}{J\omega_{r\,bas}}f_2.
\end{aligned}\right\} \quad (14)$$

Let us introduce the following generalizing coefficients:

$$a_{12} = -R_s\frac{I_{bas}}{\Psi_{bas}};\; a_{21} = \frac{R_r}{L_rL_s'}\frac{\Psi_{bas}}{I_{bas}};\; a_{22} = -\frac{R_rL_s+R_sL_r}{L_rL_s'};$$

$$c_1 = \frac{U_{bas}}{\Psi_{bas}};\; c_2 = \frac{1}{L_s'}\frac{U_{bas}}{I_{bas}};$$

$$b_{12} = \frac{3z_p}{2J}\frac{\Psi_{bas}I_{bas}}{\omega_{r\,bas}};\; b_{21} = -\frac{1}{L_s'}\frac{\Psi_{bas}\omega_{r\,bas}}{I_{bas}};$$

$$b_{22} = -\frac{R_rL_s+R_sL_r}{L_rL_s'};\; \zeta_1 = -\omega_{r\,bas};\; \zeta_2 = \omega_{bas};$$

$$\zeta_3 = -M_{bas}/(J\cdot\omega_{r\,bas});\; \zeta_4 = \omega_{r\,bas};\; \zeta_5 = -\omega_{bas}.$$

Then equations (14) will be following:

$$\left.\begin{aligned}
px_1 &= a_{12}x_2 + c_1u_x;\\
px_2 &= a_{21}x_1 + a_{22}x_2 + c_2u_x + \zeta_1y_1y_2 + \zeta_2f_1y_2;\\
py_1 &= b_{12}x_1y_2 + \zeta_3f_2;\\
py_2 &= b_{21}x_1y_1 + b_{22}y_2 + c_2u_y + \zeta_4x_2y_1 + \zeta_5f_1x_2.
\end{aligned}\right\} \quad (15)$$

The rotation speed of the coordinate system when entering relative units

$$\omega_{ks} = \frac{1}{x_1}\left(\frac{U_{bas}}{\Psi_{bas}}u_y - \frac{I_{bas}R_s}{\Psi_{bas}}y_2\right).$$

Then disturbing actions f_1 and f_2 can be represented by following expressions:

$$\left.\begin{aligned}
f_1 &= \frac{\zeta_6u_y + \zeta_7y_2}{x_1};\\
f_2 &= M_{st}/\grave{I}_{bas},
\end{aligned}\right\} \quad (16)$$

where $\zeta_6 = U_{bas}/(\Psi_{bas}\omega_{bas}) = 1;\; \zeta_7 = -I_{bas}R_s/U_{bas}$.

Thus, equations (12) and (13) allow us to execute the mathematical description of the asynchronous motor in relative units when orienting the coordinate system by vector $\mathbf{\Psi_r}$; (15) and (16) – when the coordinate system by vector $\mathbf{\Psi_s}$. The representation of asynchronous motor equations in relative units allows performing correct compensation of the dynamic properties of separate components of the control object by introducing inversed models, which are also described by differential equations in relative units. The structural block-diagram of asynchronous motor based on equations (15) and (16) is shown in following figure:

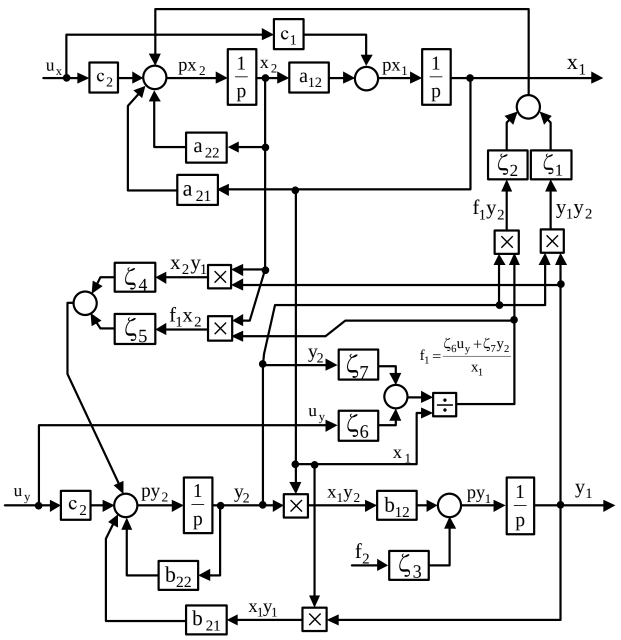

Figure 2 – The structural block-diagram of asynchronous motor in relative units when orienting the coordinate system by vector $\mathbf{\Psi_s}$

Let us represent the expressions for the coefficients of equations (12), (13) when $\mathbf{\Psi_o} = \mathbf{\Psi_r}$ and (15), (16) when $\mathbf{\Psi_o} = \mathbf{\Psi_s}$ in the form of Tabl. 1.

In order to implement the synthesis of the control system of the AC drive, it is expedient to divide the systems of equations (1) and (2) into two subsystems, which relate to reactive and active power loops. Thus, the decomposition is performed.

When orientation of the coordinate system by the vector of flux linkage $\mathbf{\Psi_o} = \mathbf{\Psi_r}$ is executed, these subsystems will have the following form:

$$\left.\begin{aligned}
&p\Psi_o = -\frac{R_r}{L_r}\Psi_o + \frac{R_rL_m}{L_r}I_{su};\\
&pI_{su} = \frac{L_mR_r}{L_r^2L_s'}\Psi_o - \frac{R_sL_r^2 + R_rL_m^2}{L_r^2L_s'}I_{su} + \rightarrow\\
&\rightarrow + I_{sv}\omega_{kr} + \frac{1}{L_s'}U_{su};
\end{aligned}\right\} \quad (17)$$

$$\left.\begin{aligned}
&p\omega = \frac{3z_pL_m}{2JL_r}I_{sv}\Psi_o - \frac{M_c}{J};\\
&pI_{sv} = -\frac{L_m}{L_rL_s'}\Psi_o\omega - \frac{R_sL_r^2 + R_rL_m^2}{L_r^2L_s'}I_{sv} + \rightarrow\\
&\rightarrow + \frac{1}{L_s'}U_{sv} - I_{su}\omega_{kr}.
\end{aligned}\right\} \quad (18)$$

In case $\mathbf{\Psi_o} = \mathbf{\Psi_s}$ we get

$$\left.\begin{aligned} p\Psi_o &= -I_{su}R_s + U_{su};\\ pI_{su} &= \frac{R_r}{L_r L_s'}\Psi_o - \frac{R_r L_s + R_s L_r}{L_r L_s'} I_{su} - \rightarrow\\ &\rightarrow - I_{sv}\left(\omega - \omega_{ks}\right) + \frac{1}{L_s'}U_{su}; \end{aligned}\right\} \quad (19)$$

$$\left.\begin{aligned} p\omega &= \frac{3z_p}{2J} I_{sv}\Psi_o - \frac{M_{st}}{J};\\ pI_{sv} &= -\frac{R_r L_s + R_s L_r}{L_r L_s'} I_{sv} - \frac{1}{L_s'}\Psi_o\omega + \rightarrow\\ &\rightarrow + I_{su}\left(\omega - \omega_{ks}\right) + \frac{1}{L_s'}U_{sv}. \end{aligned}\right\} \quad (20)$$

Table 1 – Coefficients of asynchronous motor equations in relative units

Orientation of the coordinate system by the vector of flux linkage	
rotor, $\mathbf{\Psi_o} = \mathbf{\Psi_r}$	stator, $\mathbf{\Psi_o} = \mathbf{\Psi_s}$
1	2
$a_{11} = -\frac{R_r}{L_r}$	–
$a_{12} = \frac{R_r L_m}{L_r}\frac{I_{bas}}{\Psi_{bas}}$	$a_{12} = -R_s\frac{I_{bas}}{\Psi_{bas}}$
$a_{21} = \frac{L_m R_r}{L_r^2 L_s'}\frac{\Psi_{bas}}{I_{bas}}$	$a_{21} = \frac{R_r}{L_r L_s'}\frac{\Psi_{bas}}{I_{bas}}$
$a_{22} = -\frac{R_s L_r^2 + R_r L_m^2}{L_r^2 L_s'}$	$a_{22} = -\frac{R_r L_s + R_s L_r}{L_r L_s'}$
$b_{12} = \frac{3z_p L_m}{2JL_r}\frac{I_{bas}\Psi_{bas}}{\omega_{r\,bas}}$	$b_{12} = \frac{3z_p}{2J}\frac{\Psi_{bas} I_{bas}}{\omega_{r\,bas}}$
$b_{21} = -\frac{L_m}{L_r L_s'}\frac{\omega_{r\,bas}\Psi_{bas}}{I_{bas}}$	$b_{21} = -\frac{1}{L_s'}\frac{\Psi_{bas}\omega_{r\,bas}}{I_{bas}}$
$b_{22} = -\frac{R_s L_r^2 + R_r L_m^2}{L_r^2 L_s'}$	$b_{22} = -\frac{R_r L_s + R_s L_r}{L_r L_s'}$
–	$c_1 = \frac{U_{bas}}{\Psi_{bas}}$
$c_2 = \frac{1}{L_s'}\frac{U_{bas}}{I_{bas}}$	$c_2 = \frac{1}{L_s'}\frac{U_{bas}}{I_{bas}}$
$\zeta_1 = \omega_{bas}$	$\zeta_1 = -\omega_{r\,bas}$
$\zeta_2 = -\frac{M_{bas}}{J\omega_{r\,bas}}$	$\zeta_2 = \omega_{bas}$
$\zeta_3 = -\omega_{bas}$	$\zeta_3 = -\frac{M_{bas}}{J\omega_{r\,bas}}$
$\zeta_4 = \frac{R_r L_m I_{bas}}{L_r \Psi_{bas}\omega_{bas}}$	$\zeta_4 = \omega_{r\,bas}$
$\zeta_5 = 1/z_p$	$\zeta_5 = -\omega_{bas}$
–	$\zeta_6 = 1$
–	$\zeta_7 = -\frac{I_{bas}R_s}{U_{bas}}$

The subsystems represented by equations (17)–(20) have cross feedbacks, that is the parameters of the flux linkage control loop are also included in the differential equations that describe the speed control and vice versa. At the same time, the values I_{su} and Ψ_o are controlled by flux linkage stabilization system (*FSS*), and the values I_{sv} and ω – by speed control system (*SCS*). *FSS* performs control by affecting on the stator voltage component U_{su}, and *SCS* – on the component U_{sv}.

In the construction of *FSS* and *SCS* based on the discrete time equalizer method [8], the subsystems for regulating the active and reactive powers of asynchronous motor should be considered separately.

Each of these subsystems must have a separate time equalizer tuned to a certain dynamics [9], so the cross feedbacks available in equations (17)–(20) should be regarded as perturbing and releasing them at the synthesis stage.

Equations (17)–(20) without cross feedbacks will have the following form:

$$\left.\begin{aligned} p\Psi_o &= -\frac{R_r}{L_r}\Psi_o + \frac{R_r L_m}{L_r} I_{su};\\ pI_{su} &= \frac{L_m R_r}{L_r^2 L_s'}\Psi_o - \frac{R_s L_r^2 + R_r L_m^2}{L_r^2 L_s'} I_{su} + \frac{1}{L_s'}U_{su}; \end{aligned}\right\} \quad (21)$$

$$\left.\begin{aligned} p\omega &= \frac{3z_p L_m}{2JL_r} I_{sv}\Psi_{o\,nom};\\ pI_{sv} &= -\frac{L_m}{L_r L_s'}\Psi_{o\,nom}\omega - \frac{R_s L_r^2 + R_r L_m^2}{L_r^2 L_s'} I_{sv} + \frac{1}{L_s'}U_{sv}, \end{aligned}\right\} \quad (22)$$

where $\Psi_{o\,nom} = \left|\mathbf{\Psi_o}\right| = const$ is the nominal value of the flux linkage, which is provided by FSS after motor magnetization.

$$\left.\begin{aligned} p\Psi_o &= -I_{su}R_s + U_{su};\\ pI_{su} &= \frac{R_r}{L_r L_s'}\Psi_o - \frac{R_r L_s + R_s L_r}{L_r L_s'} I_{su} + \frac{1}{L_s'}U_{su}; \end{aligned}\right\} \quad (23)$$

$$\left.\begin{aligned} p\omega &= \frac{3z_p}{2J} I_{sv}\Psi_{o\,nom};\\ pI_{sv} &= -\frac{R_r L_s + R_s L_r}{L_r L_s'} I_{sv} - \frac{1}{L_s'}\Psi_{o\,nom}\omega + \frac{1}{L_s'}U_{sv}. \end{aligned}\right\} \quad (24)$$

Equations (21) and (23) should be used for FSS synthesis and (22) and (24) – for *SCS* synthesis of the vector control system. For normalization equations (21) and (22) the relative variables (9) can be used. After substituting these relative variables equations (21) and (22) become the following:

$$\left.\begin{aligned} p\Psi_{bas}x_1 &= -\frac{R_r}{L_r}\Psi_{bas}x_1 + \frac{R_r L_m}{L_r} I_{bas}x_2;\\ pI_{bas}x_2 &= \frac{L_m R_r}{L_r^2 L_s'}\Psi_{bas}x_1 - \frac{R_s L_r^2 + R_r L_m^2}{L_r^2 L_s'} I_{bas}x_2 + \rightarrow\\ &\rightarrow + \frac{1}{L_s'}U_{bas}u_x; \end{aligned}\right\}$$

$$\left.\begin{aligned} &p\omega_{r\,bas}y_1=\frac{3z_pL_m}{2JL_r}\Psi_{o\,nom}I_{bas}y_2;\\ &pI_{bas}y_2=-\frac{L_m}{L_rL_s'}\Psi_{o\,nom}\omega_{r\,bas}y_1-\rightarrow\\ &\rightarrow-\frac{R_sL_r^2+R_rL_m^2}{L_r^2L_s'}I_{bas}y_2+\frac{1}{L_s'}U_{bas}u_y. \end{aligned}\right\}$$

Let us transform the obtained equation by highlighting in their left parts components px_1, px_2, py_1 and py_2

$$\left.\begin{aligned} &px_1=-\frac{R_r}{L_r}x_1+\frac{R_rL_m}{L_r}\frac{I_{bas}}{\Psi_{bas}}x_2;\\ &px_2=\frac{L_mR_r}{L_r^2L_s'}\frac{\Psi_{bas}}{I_{bas}}x_1-\frac{R_sL_r^2+R_rL_m^2}{L_r^2L_s'}x_2+\frac{1}{L_s'}\frac{U_{bas}}{I_{bas}}u_x; \end{aligned}\right\}$$

$$\left.\begin{aligned} &py_1=\frac{3z_pL_m}{2JL_r}\frac{I_{bas}}{\omega_{r\,bas}}\Psi_{o\,nom}y_2;\\ &py_2=-\frac{L_m}{L_rL_s'}\frac{\omega_{r\,bas}}{I_{bas}}\Psi_{o\,nom}y_1-\rightarrow\\ &\rightarrow-\frac{R_sL_r^2+R_rL_m^2}{L_r^2L_s'}y_2+\frac{1}{L_s'}\frac{U_{bas}}{I_{bas}}u_y, \end{aligned}\right\}$$

and also introduce the generalizing coefficients

$$\left.\begin{aligned} &px_1=a_{11}x_1+a_{12}x_2;\\ &px_2=a_{21}x_1+a_{22}x_2+c_2u_x, \end{aligned}\right\} \qquad (25)$$

where $c_2=\frac{1}{L_s'}\frac{U_{bas}}{I_{bas}}$; $a_{11}=-\frac{R_r}{L_r}$; $a_{12}=\frac{R_rL_m}{L_r}\frac{I_{bas}}{\Psi_{bas}}$; $a_{21}=\frac{L_mR_r}{L_r^2L_s'}\frac{\Psi_{bas}}{I_{bas}}$; $a_{22}=-\frac{R_sL_r^2+R_rL_m^2}{L_r^2L_s'}$;

$$\left.\begin{aligned} &py_1=b_{12}y_2;\\ &py_2=b_{21}y_1+b_{22}y_2+c_2u_y, \end{aligned}\right\} \qquad (26)$$

where $c_2=\frac{1}{L_s'}\frac{U_{bas}}{I_{bas}}$; $b_{12}=\frac{3z_pL_m}{2JL_r}\frac{I_{bas}}{\omega_{r\,bas}}\Psi_{o\,nom}$; $b_{21}=-\frac{L_m}{L_rL_s'}\frac{\omega_{r\,bas}}{I_{bas}}\Psi_{o\,nom}$; $b_{22}=-\frac{R_sL_r^2+R_rL_m^2}{L_r^2L_s'}$.

The calculation formulas for the generalizing coefficients a_{11}, a_{12}, a_{21}, a_{22}, b_{22} and c_2 are completely coincide with the formulas obtained for the calculation of the coefficients in the equations (12) for the asynchronous motor model in relative units with coordinate system orientation by vector $\mathbf{\Psi_r}$. The coefficients b_{12} and b_{21} have other formulas for calculation, since linearization in equations (18) assumes that the rotor flux linkage is a constant value $\Psi_{o\,nom}=|\mathbf{\Psi_o}|=const$.

For normalization equations (23) and (24) the relative variables (9) can be used.

$$\left.\begin{aligned} &p\Psi_{bas}x_1=-R_sI_{bas}x_2+U_{bas}u_x;\\ &pI_{bas}x_2=\frac{R_r}{L_rL_s'}\Psi_{bas}x_1-\frac{R_rL_s+R_sL_r}{L_rL_s'}I_{bas}x_2+\rightarrow\\ &\rightarrow+\frac{1}{L_s'}U_{bas}u_x; \end{aligned}\right\}$$

$$\left.\begin{aligned} &p\omega_{r\,bas}y_1=\frac{3z_p}{2J}\Psi_{o\,nom}I_{bas}y_2;\\ &pI_{bas}y_2=-\frac{1}{L_s'}\Psi_{o\,nom}\omega_{r\,bas}y_1-\rightarrow\\ &\rightarrow-\frac{R_rL_s+R_sL_r}{L_rL_s'}I_{bas}y_2+\frac{1}{L_s'}U_{bas}u_y. \end{aligned}\right\}$$

Let us transform the obtained equation by highlighting in their left parts components px_1, px_2, py_1 and py_2

$$\left.\begin{aligned} &px_1=-R_s\frac{I_{bas}}{\Psi_{bas}}x_2+\frac{U_{bas}}{\Psi_{bas}}u_x;\\ &px_2=\frac{R_r}{L_rL_s'}\frac{\Psi_{bas}}{I_{bas}}x_1-\frac{R_rL_s+R_sL_r}{L_rL_s'}x_2+\frac{1}{L_s'}\frac{U_{bas}}{I_{bas}}u_x; \end{aligned}\right\}$$

$$\left.\begin{aligned} &py_1=\frac{3z_p}{2J}\frac{I_{bas}}{\omega_{r\,bas}}\Psi_{o\,nom}y_2;\\ &py_2=-\frac{1}{L_s'}\frac{\omega_{r\,bas}}{I_{bas}}\Psi_{o\,nom}y_1-\frac{R_rL_s+R_sL_r}{L_rL_s'}y_2+\rightarrow\\ &\rightarrow+\frac{1}{L_s'}\frac{U_{bas}}{I_{bas}}u_y, \end{aligned}\right\}$$

and also introduce the generalizing coefficients

$$\left.\begin{aligned} &px_1=a_{12}x_2+c_1u_x;\\ &px_2=a_{21}x_1+a_{22}x_2+c_2u_x, \end{aligned}\right\} \qquad (27)$$

where $c_1=\frac{U_{bas}}{\Psi_{bas}}$; $c_2=\frac{1}{L_s'}\frac{U_{bas}}{I_{bas}}$; $a_{12}=-R_s\frac{I_{bas}}{\Psi_{bas}}$; $a_{21}=\frac{R_r}{L_rL_s'}\frac{\Psi_{bas}}{I_{bas}}$; $a_{22}=-\frac{R_rL_s+R_sL_r}{L_rL_s'}$;

$$\left.\begin{aligned} &py_1=b_{12}y_2;\\ &py_2=b_{21}y_1+b_{22}y_2+c_2u_y, \end{aligned}\right\} \qquad (28)$$

where $c_2=\frac{1}{L_s'}\frac{U_{bas}}{I_{bas}}$; $b_{12}=\frac{3z_p}{2J}\frac{I_{bas}}{\omega_{r\,bas}}\Psi_{o\,nom}$; $b_{21}=-\frac{1}{L_s'}\frac{\omega_{r\,bas}}{I_{bas}}\Psi_{o\,nom}$; $b_{22}=-\frac{R_rL_s+R_sL_r}{L_rL_s'}$.

The calculation formulas for the generalizing coefficients a_{12}, a_{21}, a_{22}, b_{22}, c_1, c_2 are completely coincide with the formulas obtained for the calculation of the coefficients in the equations (15) for the asynchronous motor model in relative units with coordinate system orientation by vector $\mathbf{\Psi_s}$. The coefficients b_{12} and b_{21} have other formulas for calculation, since linearization in equations (20) assumes that the stator flux linkage is a constant value $\Psi_{o\,nom} = |\mathbf{\Psi_o}| = const$.

Using equation (27), let us draw up the structural block-diagram of reactive power loop on the basis of the above assumptions:

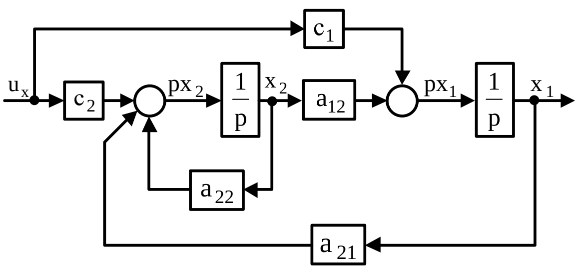

Figure 3 – The structural block-diagram of reactive power loop in relative units with coordinate system orientation by vector $\mathbf{\Psi_s}$

Since equations (28) differ from equations (26) only by the formulas for calculating the coefficients, the structural block-diagram of the active power loop in relative units with coordinate system orientation by vector $\mathbf{\Psi_s}$ will be the same for $\mathbf{\Psi_o} = \mathbf{\Psi_r}$.

Let us represent the expressions for the coefficients of equations (25), (26) when $\mathbf{\Psi_o} = \mathbf{\Psi_r}$ and (27), (28) when $\mathbf{\Psi_o} = \mathbf{\Psi_s}$ in the form of Tabl. 2.

Table 2 – Coefficients of asynchronous motor linearized equations in relative units

Orientation of the coordinate system by the vector of flux linkage	
rotor, $\mathbf{\Psi_o} = \mathbf{\Psi_r}$	stator, $\mathbf{\Psi_o} = \mathbf{\Psi_s}$
$a_{11} = -R_r / L_r$	–
$a_{12} = \frac{R_r L_m}{L_r} \frac{I_{bas}}{\Psi_{bas}}$	$a_{12} = -R_s \frac{I_{bas}}{\Psi_{bas}}$
$a_{21} = \frac{L_m R_r}{L_r^2 L_s'} \frac{\Psi_{bas}}{I_{bas}}$	$a_{21} = \frac{R_r}{L_r L_s'} \frac{\Psi_{bas}}{I_{bas}}$
$a_{22} = -\frac{R_s L_r^2 + R_r L_m^2}{L_r^2 L_s'}$	$a_{22} = -\frac{R_r L_s + R_s L_r}{L_r L_s'}$
$b_{12} = \frac{3 z_p L_m}{2 J L_r} \frac{I_{bas}}{\omega_{r\,bas}} \Psi_{o\,nom}$	$b_{12} = \frac{3 z_p}{2J} \frac{I_{bas}}{\omega_{r\,bas}} \Psi_{o\,nom}$
$b_{12} = \frac{3 z_p L_m}{2 J L_r} \frac{I_{bas}}{\omega_{r\,bas}} \Psi_{o\,nom}$	$b_{12} = \frac{3 z_p}{2J} \frac{I_{bas}}{\omega_{r\,bas}} \Psi_{o\,nom}$
$b_{21} = -\frac{L_m}{L_r L_s'} \frac{\omega_{r\,bas}}{I_{bas}} \Psi_{o\,nom}$	$b_{21} = -\frac{1}{L_s'} \frac{\omega_{r\,bas}}{I_{bas}} \Psi_{o\,nom}$
$b_{22} = -\frac{R_s L_r^2 + R_r L_m^2}{L_r^2 L_s'}$	$b_{22} = -\frac{R_r L_s + R_s L_r}{L_r L_s'}$
–	$c_1 = U_{bas} / \Psi_{bas}$
$c_2 = (1/L_s')(U_{bas}/I_{bas})$	$c_2 = (1/L_s')(U_{bas}/I_{bas})$

CONCLUSIONS. The given differential equations and structural block-diagram of the asynchronous motor in two-phase orthogonal coordinate systems oriented by flux linkage vectors provide an exhaustive mathematical description necessary for the implementation of the synthesis of automated electromechanical systems based on a discrete time equalizer [10].

Expressions for the coefficients of the asynchronous motor equations in relative units, obtained as a result of the transformations performed and entered into a generalization table, allow us to calculate the parameters of the structural block-diagram and mathematical models for a wide range of industrial asynchronous motors.

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ДЕКОМПОЗИЦИЯ И ЛИНЕАРИЗАЦИЯ МАТЕМАТИЧЕСКИХ МОДЕЛЕЙ АСИНХРОННОГО ДВИГАТЕЛЯ В ОТНОСИТЕЛЬНЫХ ЕДИНИЦАХ

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Для выполнения синтеза современных автоматизированных электромеханических систем с асинхронными двигателями применяют двухфазные математические модели, которые адекватно отражают процессы, протекающие в реальной машине. Предложенные в работе математические преобразования и численные расчеты выполнены с применением математических моделей асинхронного двигателя в двухфазных ортогональных системах координат, ориентированных по векторам потокосцепления ротора и статора. Для обеспечения возможности синтеза систем управления, строящихся на принципах решения обратных задач динамики, использована система относительных единиц с равными взаимными индуктивностями между статорными и роторными обмотками, а также между фазными обмотками ротора. Асинхронный двигатель условно разделен на два объекта управления: для контура реактивной и контура активной мощности. Определение моделей объектов управления осуществлено с применением методов декомпозиции и линеаризации систем дифференциальных уравнений. Полученные коэффициенты линеаризованных уравнений асинхронного двигателя в относительных единицах позволяют корректно компенсировать динамические свойства отдельных составляющих объекта управления автоматизированной электромеханической системы с помощью обратных динамических моделей.

Ключевые слова: декомпозиция, линеаризация, асинхронный двигатель.

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